2021 Digital Exam Practice

AP Statistics

Scoring Materials for Digital Exam Practice

Please note: the digital exam practice resource was developed for students to complete technology checks, experience the digital platform, and practice answering exam questions, including each type of multiple-choice and free-response question they will encounter on exam day.

This digital exam practice is not a full-length exam, and it does not represent the complete scope of content and skills that students will see on the actual AP exam. This digital exam practice includes only content that would typically be taught in the first half of the school year, following the unit sequence in the AP Statistics Course and Exam Description. For more information on the 2021 Exam format, please visit: apcentral.collegeboard.org/pdf/ap-2021-exam-formats.pdf

AP Exams are scored differently than traditional high school or college exams. When an AP Exam is administered, psychometric analysis determines the score ranges corresponding with each AP Exam score (5, 4, 3, 2, and 1) based on a composite score scale that combines and weights the different exam parts. Earning 40-50% of the available points can result in a score of 3 or better on many AP Exams. However, because the number of points corresponding with each AP Exam score can vary on different exams, students and teachers should not use the results of the digital exam practice to predict performance on the 2021 AP Exam.

Multiple-Choice Answer Key

Multiple-Choice Question	Answer
1	D
2	E
3	B
4	E
5	E
6	A
7	A
8	B
9	C

Question 1: Focus on Exploring Data — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question and the scoring criteria identify the specific components of the model solution that are used to determine the score.
- Because responses are typed, minor typographical errors and/or omissions in mathematical expressions (e.g., missing parentheses, unformatted superscripts/subscripts, etc.) do not disqualify a response from earning a particular score, provided the response is otherwise correct and clear.
- Examples of acceptable typed expressions are included with the model solution, where applicable.

	Model Solution	Scoring
(a)	The median is the value with half of the P-T ratios at or below it and half of the values at or above it. For n observations in a group, use $(n + 1) / 2$ to find the position of the median in the ordered list of observations. For states west of the Mississippi ($n = 24$) the median falls between the 12th and 13th value in the ordered list, and both the 12th and 13th values fall in the interval 15–16. For states east of the Mississippi ($n = 26$) the median falls between the 13th and 14th value in the ordered list, and both of these values also fall in the interval 15–16. From the histogram, cumulative frequencies for the two groups are below: For interval 12-13, west is 1 and east is 2. For interval 13-14, west is $1 + 4 = 5$ and east is $2 + 4 = 6$. For interval 14-15, west is $1 + 4 + 6 = 11$ and east is $2 + 4 + 4 = 10$. For interval 15-16, west is $1 + 4 + 6 + 3 = 14$ and east is $2 + 4 + 4 + 11 = 21$. Thus, the median P-T ratio for both groups is at least 15 students per teacher and at most 16 students per teacher.	**Essentially correct (E)** if the response describes a correct estimation method AND provides appropriate estimates (i.e., values between 15 and 16, inclusive). **Partially correct (P)** if the response describes a correct estimation method but does not provide the estimates *OR* if the response describes a method that conveys the idea of median as the middle value but is not entirely correct (e.g., it describes the 12th value rather than the average of the 12th and 13th values) AND provides reasonable estimates *OR* if the response provides an incomplete description of the method AND provides appropriate estimates (i.e., values between 15 and 16, inclusive) *OR* if the response shows work only for the histograms AND correct estimates are provided but no verbal explanation of the method is given. **Incorrect (I)** if the response does not meet the criteria for E or P.

	Model Solution	Scoring
(b)	The shapes of the two histograms are different. The histogram for states that are west of the Mississippi River is unimodal and skewed to the right, whereas the histogram for states that are east of the Mississippi River is unimodal and nearly symmetric. As noted in part (a), the medians of the two distributions are about the same, between 15 and 16 for both distributions. The histograms also show that there is more variability in the P-T ratios for states that are west of the Mississippi River. Although the greatest and least values for each group are not known, the range can be approximated. The range for the west is at most 22 – 12 = 10, and the range for the east is at most 19 – 12 = 7.	**Essentially correct (E)** if the response provides appropriate comparative statements for the centers, the shapes, and the spreads of the two groups. **Partially correct (P)** if the response provides appropriate comparative statements for only two of the three characteristics (center, shape, and spread) *OR* if the response does not provide appropriate comparative statements but provides correct information regarding all three characteristics (center, shape, and spread) for both groups. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- The shape of the east histogram can be described as skewed, *approximately* symmetric, or *approximately* normal. However, if the shape is described as symmetric or normal, the response cannot be scored E.
- If a comparative statement about medians is made in part (a) or in part (c), the statement satisfies the comparison of center in part (b).

	Model Solution	Scoring
(c)	The medians of the two distributions are about the same, as determined in part (a). The distribution of P-T ratios for states that are west of the Mississippi River is skewed to the right, indicating that the mean will probably be higher than the median. The rough symmetry for the east group indicates that the mean will be close to the median. Thus, the mean for the west group will probably be greater than the mean for the east group.	**Essentially correct (E)** if the response indicates that the west group has a higher mean than the east group AND provides a justification based on the relationship between means and medians for distributions with different shapes. **Partially correct (P)** if the response indicates that the west group has a higher mean but provides a justification that is not based on the relationship between means and medians for distributions with different shapes, e.g., the justification is not based on the answers to parts (a) and (b) *OR* if the response includes correct statements about the relative sizes of the mean and median for each group but does not explicitly compare the means of the two groups *OR* if the response provides a justification based on the relationship between the mean and median for a skewed distribution but concludes that the mean will be smaller than the median for a right-skewed distribution. **Incorrect (I)** if the response indicates either group with no justification *OR* if the response does not otherwise meet the criteria for E or P.

Scoring for Question 1	Score
Complete Response Three parts essentially correct	**4**
Substantial Response Two parts essentially correct and one part partially correct	**3**
Developing Response Two parts essentially correct and no part partially correct *OR* One part essentially correct and one or two parts partially correct *OR* Three parts partially correct	**2**
Minimal Response One part essentially correct *OR* No part essentially correct and two parts partially correct	**1**

Question 2: Focus on Sampling and Experimentation — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question and the scoring criteria identify the specific components of the model solution that are used to determine the score.
- Because responses are typed, minor typographical errors and/or omissions in mathematical expressions (e.g., missing parentheses, unformatted superscripts/subscripts, etc.) do not disqualify a response from earning a particular score, provided the response is otherwise correct and clear.
- Examples of acceptable typed expressions are included with the model solution, where applicable.

	Model Solution	Scoring
(a)	Treatments: Sprays with four different concentrations of the fungus (0 ml/L, 1.25 ml/L, 2.5 ml/L, and 3.75 ml/L) Experimental units: 20 containers, each containing the same number of insects Response variable: Number of insects that are still alive in each container one week after spraying	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies the 4 concentrations (or mixtures or sprays) as the treatments 2. Identifies the 20 containers as the experimental units 3. Identifies the number of insects that are still alive in each container as the response variable **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Listing the four treatments satisfies component 1 (including ml/L is not required). However, if the list does not include all four treatments, component 1 is not satisfied.
- To satisfy component 1, the response must refer to plural concentrations/mixtures/sprays (e.g., the mixtures, the levels of concentration). Referring only to the explanatory variable (concentration) does not satisfy component 1.
- The following responses satisfy component 2: "the 20 containers"; "the containers"; "the 20 groups of insects"; or "the groups of insects in each container." References to only "groups of insects" do not satisfy component 2 because it is unclear if these groups are formed by treatment or by container.
- To satisfy component 3, it must be clear that the response variable is being measured separately for each experimental unit. A response that says only "number of insects alive" does not satisfy component 3 because it could be referring to the total number of insects alive.

- To satisfy component 3, the response must be stated as a variable by using "number of" or equivalent. For example, "insects alive in each container" is not a variable and would not satisfy component 3.

Additional Notes (continued):

- If the response states that the insects are the experimental units, then component 3 can still be satisfied by providing a binary response variable for each insect (e.g., whether the insect lived or died, survival status).

	Model Solution	Scoring
(b)	Yes. Because the 0 ml/L concentration contains no fungus, the containers that are sprayed with the 0 ml/L concentration form the control group.	**Essentially correct (E)** if the response indicates that there is no control group AND justifies this claim by identifying the control group or by explaining that there is a treatment which contains no fungus. **Partially correct (P)** if the response indicates that there is no control group because every container is sprayed with some mixture *OR* if the response states that there is a control group but implies that 0 ml/L is not a treatment (e.g., "the containers with 0 ml/L form a control group because they don't receive a treatment"; "yes, there is a group that got no treatment"). **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- The response does not need to explain the purpose of a control group.
- The response does not need to explicitly say "yes" – it can be implied by stating that there is a control group or saying "the control group is …."

Model Solution	Scoring
(c) Label each container with a unique integer from 1 to 20. Then use a random number generator to choose 15 integers from 1 to 20 without replacement. Use the first five of these numbers to identify the five containers that will receive the 0 ml/L treatment. Use the second five of these numbers to identify the five containers that will receive the 1.25 ml/L treatment. Use the third five of these numbers to identify the five containers that will receive the 2.5 ml/L treatment. The remaining five containers will receive the 3.75 ml/L treatment. *Alternative Solution:* Using equally sized slips of paper, label five slips with 0 ml/L, five slips with 1.25 ml/L, five slips with 2.5 ml/L, and five slips with 3.75 ml/L. Mix the slips of paper in a hat. For each container, select a slip of paper from the hat (without replacement) and spray that container with the treatment selected.	**Essentially correct (E)** if the response satisfies the following three components: 1. Creates appropriate labels for the units/treatments (e.g., label the containers from 1 through 20, label 20 slips of paper with five for each treatment) 2. Describes how to correctly implement the random assignment process 3. The random assignment process results in an equal number of experimental units assigned to each treatment **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- If the response states that insects are the experimental units in part (a), the response in part (c) can be in terms of insects or containers. In either case, the same three components are used to determine the score.
- If the response states that the containers are the experimental units in part (a), but only describes how to assign insects to treatments in part (c), component 1 is not satisfied.
- For responses that use slips of paper:
  - If the number of slips of paper is not equal to the number of experimental units, then component 1 is not satisfied. The slips of paper do not need to be specifically identified as equally-sized.
  - If the slips of paper are not mixed/shuffled or the slips are not "selected at random," component 2 is not satisfied. Sampling without replacement is implied when using slips of paper, unless the response specifies sampling with replacement.
- For responses that use random number generators (or a 20-sided die):
  - If the initial assignment of numbers to units does not give each unit the same probability of being assigned to each treatment (e.g., units are represented by different numbers of integers), then component 1 is not satisfied.
  - If the response does not indicate that the numbers are selected without replacement or that different numbers must be used, the response does not satisfy component 2. The response does not need to specify the interval of numbers from which they are selecting (e.g., randomly generate a number from 1 to 20).

Additional Notes (continued):

- For responses that use a table of random digits:
  - If the initial assignment of numbers to units does not give each unit the same probability of being assigned to each treatment, component 1 is not satisfied. For example, responses that use the labels 1 to 20 (not 01 to 20) do not satisfy component 1 because label 1 has a $\frac{1}{10}$ probability of being selected but label 20 has a $\frac{1}{100}$ probability of being selected.
  - If the response does not indicate that the numbers are selected without replacement or that different numbers must be used, the response does not satisfy component 2. The response does not need to specify the interval of numbers from which they are selecting or state that the numbers corresponding to unused labels will be skipped (e.g., skip numbers 00 and 21 to 99).
- For responses that use a 4-sided die (or random integers from 1 to 4):
  - If the die is rolled for each experimental unit, then component 3 is not satisfied because an equal number of units per treatment is not guaranteed.
  - If the die is rolled for each experimental unit until treatments are "full," then component 1 is not satisfied because this setup doesn't allow for all possible random assignments to be equally likely (unless the order of the units is randomized initially).
- If a response groups the experimental units before any random assignment (e.g., forms five groups of four containers or four groups of five containers), and then randomly assigns treatments to the groups or randomly assigns treatments within each group, component 1 is not satisfied. However, if a response forms groups in the context of a randomized block design with a reasonable blocking variable, component 1 can be satisfied.
- If a response describes two different random assignment processes in detail (e.g., how to randomly assign insects to containers and how to assign containers to treatments), both descriptors are scored according to the three components and the lower score is used.
- Responses that assign experimental units only to groups and not to treatments (e.g., randomly select five containers and put them in group 1) do not satisfy component 3.
- If the response randomly assigns insects to containers, the containers must be assigned to a treatment to satisfy component 3. In this case, the assignment of treatment to container does not need to be at random to satisfy component 3.

Scoring for Question 2	Score
Complete Response Three parts essentially correct	**4**
Substantial Response Two parts essentially correct and one part partially correct	**3**
Developing Response Two parts essentially correct and no part partially correct *OR* One part essentially correct and one or two parts partially correct *OR* Three parts partially correct	**2**
Minimal Response One part essentially correct *OR* No part essentially correct and two parts partially correct	**1**

Question 3: Focus on Probability and Simulation — 4 points

General Scoring Notes

- Parts (a), (b), and (c) of the question are initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). Part (d) of the question is initially scored by determining if it meets the criteria for essentially correct (E) or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question and the scoring criteria identify the specific components of the model solution that are used to determine the score.
- Because responses are typed, minor typographical errors and/or omissions in mathematical expressions (e.g., missing parentheses, unformatted superscripts/subscripts, etc.) do not disqualify a response from earning a particular score, provided the response is otherwise correct and clear.
- Examples of acceptable typed expressions are included with the model solution, where applicable.

	Model Solution	Scoring
(a)	X is not binomial since the trials are not independent and the conditional probabilities of selecting a male change at each trial depending on the previous outcome(s), due to the sampling without replacement.	**Essentially correct (E)** if the response indicates that trials are not independent, with an explanation that independence means the outcome on any trial will not impact the probability of success on future trials *OR* if the response indicates that the probability of selecting a male on any given trial depends on the results of previous trials. **Partially correct (P)** if the response indicates that trials are not independent but provides an incomplete explanation of independence (e.g., simply stating that the "probability changes") *OR* if the response indicates that there is sampling without replacement but does not provide an explanation of the impact on independence or the probability of success. **Incorrect (I)** if the response does not meet the criteria for E or P.

	Model Solution	Scoring
(b)	$P(X = 4) = (10/20)(9/19)(8/18)(7/17)$ $= 0.043$ **Examples of acceptable typed expressions:** • 10/20*9/19*8/18*7/17 • 10/20 x 9/19 x 8/18 x 7/17 *Alternative Solution:* $P(X = 4) = (C(10,4) * C(10,0)) / C(20,4)$ $= 0.043$ **Examples of acceptable typed expressions:** • (10 4) * (10 0) / (20 4) • 10C4 * 10C0 / 20C4 • Binomial coefficient for n = 10, k = 4 multiplied by binomial coefficent for n = 10, k = 0 divided by binomial coefficient for n = 20, k = 4	**Essentially correct (E)** if the response provides a correctly computed probability (with minor arithmetic errors overlooked) AND includes supporting work or rationale. **Partially correct (P)** if the response provides a correctly computed probability but includes incomplete justification (e.g., does not provide supporting work). **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that only provides the product of fractions, without the decimal approximation, is scored E.

	Model Solution	Scoring
(c)	No. If males and females were equally represented, the probability of observing four males is small (0.043).	**Essentially correct (E)** if the response states that there was not equal representation AND provides a correct interpretation of the probability computed in part (b). **Partially correct (P)** if the response states that there was not equal representation but provides an incomplete interpretation of the probability computed in part (b) (e.g., stating "the probability of observing four males is small" without including the conditional statement) *OR* if the response states that there was not equal representation but provides a correct interpretation of an ambiguous probability or a different probability than the probability computed in part (b) *OR* if the response provides a correct interpretation of the probability computed in part (b) but states that there was equal representation. **Incorrect (I)** if the response does not meet the criteria for E or P.

	Model Solution	Scoring
(d)	No, we cannot generalize to the population of all brontosaurs because it is not reasonable to regard this sample as a random sample from the population of all brontosaurs; there is reason to suspect that this sampling method might cause bias.	**Essentially correct (E)** if the response indicates that generalization is not possible AND provides a justification that the sample cannot be viewed as a random sample of all brontosaurs or that there is reason to suspect that this sample might not be representative of the population at large. **Partially correct (P)** if the response indicates that generalization is not possible but provides an incomplete justification (e.g., "the sample is not random"). **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Discussions about the conditions for inference are irrelevant and ignored in scoring part (d).

Scoring for Question 3
Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.

	Score
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

Question 4: Investigative Task — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question and the scoring criteria identify the specific components of the model solution that are used to determine the score.
- Because responses are typed, minor typographical errors and/or omissions in mathematical expressions (e.g., missing parentheses, unformatted superscripts/subscripts, etc.) do not disqualify a response from earning a particular score, provided the response is otherwise correct and clear.
- Examples of acceptable typed expressions are included with the model solution, where applicable.

	Model Solution	Scoring
(a)	For a car with length 175 inches, the predicted value for the car's FCR, based on the least squares regression line, is predicted FCR = –1.596 + 0.037(175) = 4.92 gallons per 100 miles. The actual FCR for the car is 5.88, so the residual is 5.88 – 4.92 = 0.96. The residual value means that the car's FCR is 0.96 gallons per 100 miles greater than would be predicted for a car of its length.	**Essentially correct (E)** if the response satisfies the following two components: 1. A correct residual value with supporting calculation 2. A correct interpretation of the residual value, using sufficient context **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- If the residual value is incorrect, the interpretation should be considered correct if it follows from the incorrect residual value.
- To include sufficient context, the response must state the response variable ("FCR"), the units of the response variable ("gallons per 100 miles"), and the explanatory variable ("length").
- Correct interpretation of the residual value must include the correct direction and magnitude of the FCR value away from the predicted FCR value.
- A calculated residual value which is slightly different from 0.96 due to the number of significant digits is acceptable.

	Model Solution	Scoring
(b)	**(i)** The coordinates of the point on graph III that corresponds to the car represented by point A on graphs I and II are (93, 0.96). **(ii)** Point B corresponds to a car with an actual FCR that is very close to the FCR that would be predicted for a car with its length by the regression model which predicts FCR using the explanatory variable length.	**Essentially correct (E)** if the response satisfies the following two components: 1. Correctly identifies the coordinates of the point on graph III 2. Provides a reasonable interpretation of the car associated with point B having a residual near 0 that refers to predicting FCR based on length **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A correct response for component 2 must include reference to the observed FCR value of the car represented by point B, not the point B itself.
- The response to part (i) and part (ii) need not be separated or individually labeled.

	Model Solution	Scoring
(c)	Graph II reveals a moderate association that is positive and linear. In contrast, there is a weak association that is positive and linear in graph III. The association between engine size and residual (from predicting FCR based on length) is stronger than the association between wheel base and residual (from predicting FCR based on length).	**Essentially correct (E)** if the response satisfies the following three components: 1. A description of form AND direction for both graphs 2. A description of the strength of association for both graphs 3. A comparison between the two graphs **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Part (c) is focused on the comparison of graph II and graph III. Inferences drawn from patterns in these graphs are considered in part (d).
- Linear is needed for form in graph II.
- Graph III may be described as having no association between wheel base and the residuals of FCR based on length, which is sufficient for describing the form, direction and strength of association of graph III.

	Model Solution	Scoring
(d)	Engine size is a better choice than wheel base for including with length in a regression model for predicting FCR. The stronger association between engine size and residual (from predicting FCR based on length) indicates that engine size is more useful than wheel base for reducing the variability in FCR values that remains unexplained (as indicated by residuals) after predicting FCR based on length.	**Essentially correct (E)** if the response indicates that engine size is the better choice AND provides a justification based on the following two components: 1. The strong(er) association 2. Reducing the variability that remains unexplained in the model which predicts FCR based on length **Partially correct (P)** if the response indicates that engine size is the better choice AND provides a justification based on only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Describing the variables in graph II and graph III as residuals is not required but can be used positively in holistic scoring. Incorrect descriptions of graph II or graph III or the variables in graphs are not acceptable.

Scoring for Question 4
Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.

	Score
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.