



**INCLUDES**

- ✓ Course framework
- ✓ Sample exam questions
- ✓ Scoring guidelines

# AP<sup>®</sup> Statistics

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**COURSE AND EXAM DESCRIPTION**

Effective  
**Fall 2026**



# AP<sup>®</sup> Statistics

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## COURSE AND EXAM DESCRIPTION

Effective  
**Fall 2026**

**AP COURSE AND EXAM DESCRIPTIONS ARE UPDATED PERIODICALLY**

Please visit [AP Central](#) to determine whether a more recent course and exam description is available.

# What AP® Stands For

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Thousands of Advanced Placement teachers have contributed to the principles articulated here. These principles are not new; they are, rather, a reminder of how AP already works in classrooms nationwide. The following principles are designed to ensure that teachers' expertise is respected, required course content is understood, and students are academically challenged and free to make up their own minds.

1. AP stands for clarity and transparency. Teachers and students deserve clear expectations. The Advanced Placement Program makes public its course frameworks and sample assessments. Confusion about what is permitted in the classroom disrupts teachers and students as they navigate demanding work.
2. AP is an unflinching encounter with evidence. AP courses enable students to develop as independent thinkers and to draw their own conclusions. Evidence and the scientific method are the starting place for conversations in AP courses.
3. AP opposes censorship. AP is animated by a deep respect for the intellectual freedom of teachers and students alike. If a school bans required topics from their AP courses, the AP Program removes the AP designation from that course and its inclusion in the AP Course Ledger provided to colleges and universities. For example, the concepts of evolution are at the heart of college biology, and a course that neglects such concepts does not pass muster as AP Biology.
4. AP opposes indoctrination. AP students are expected to analyze different perspectives from their own, and no points on an AP Exam are awarded for agreement with any specific viewpoint. AP students are not required to feel certain ways about themselves or the course content. AP courses instead develop students' abilities to assess the credibility of sources, draw conclusions, and make up their own minds.

As the AP English Literature course description states: "AP students are not expected or asked to subscribe to any one specific set of cultural or political values, but are expected to have the maturity to analyze perspectives different from their own and to question the meaning, purpose, or effect of such content within the literary work as a whole."

5. AP courses foster an open-minded approach to the histories and cultures of different peoples. The study of different nationalities, cultures, religions, races, and ethnicities is essential within a variety of academic disciplines. AP courses ground such studies in primary sources so that students can evaluate experiences and evidence for themselves.
6. Every AP student who engages with evidence is listened to and respected. Students are encouraged to evaluate arguments but not one another. AP classrooms respect diversity in backgrounds, experiences, and viewpoints. The perspectives and contributions of the full range of AP students are sought and considered. Respectful debate of ideas is cultivated and protected; personal attacks have no place in AP.
7. AP is a choice for parents and students. Parents and students freely choose to enroll in AP courses. Course descriptions are available online for parents and students to inform their choice. Parents do not define which college-level topics are suitable within AP courses; AP course and exam materials are crafted by committees of professors and other expert educators in each field. AP courses and exams are then further validated by the American Council on Education and studies that confirm the use of AP scores for college credits by thousands of colleges and universities nationwide.

The AP Program encourages educators to review these principles with parents and students so they know what to expect in an AP course. Advanced Placement is always a choice, and it should be an informed one. AP teachers should be given the confidence and clarity that once parents have enrolled their child in an AP course, they have agreed to a classroom experience that embodies these principles.

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# About AP

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The Advanced Placement® Program (AP®) enables willing and academically prepared students to pursue college-level studies—with the opportunity to earn college credit, advanced placement, or both—while still in high school. Through AP courses in 40 subjects, each culminating in a challenging exam, students learn to think critically, construct solid arguments, and see many sides of an issue—skills that prepare them for college and beyond. Taking AP courses demonstrates to college admission officers that students have sought the most challenging curriculum available to them, and research indicates that students who score a 3 or higher on an AP Exam typically experience greater academic success in college and are more likely to earn a college degree than non-AP students. Each AP teacher’s syllabus is evaluated and approved by faculty from some of the nation’s leading colleges and universities, and AP Exams are developed and scored by college faculty and experienced AP teachers. Most four-year colleges and universities in the United States grant credit, advanced placement, or both on the basis of successful AP Exam scores—more than 3,300 institutions worldwide annually receive AP scores.

## AP Course Development

In an ongoing effort to maintain alignment with best practices in college-level learning, AP courses and exams emphasize challenging, research-based curricula aligned with higher education expectations.

Individual teachers are responsible for designing their own curriculum for AP courses, selecting appropriate college-level readings, assignments, and resources. This course and exam description presents the content and skills that are the focus of the corresponding college course and that appear on the AP Exam. It also organizes the content and skills into a series of units that represent a sequence found in widely adopted college textbooks and that many AP teachers have told us they follow to focus their instruction. The intention of this publication is to respect teachers’ time and expertise by providing a roadmap that they can modify and adapt to their local priorities and preferences. Moreover, by organizing the AP course content and skills into units, the AP Program is able to provide teachers and students with free formative assessments—Progress Checks—that teachers can assign throughout the year to measure student progress as they acquire content knowledge and develop skills.

## Enrolling Students: Access, Opportunity, and Readiness

The AP Program welcomes all students willing to challenge themselves with college-level coursework and career preparation. We strongly encourage educators to invite students into AP classes, including students from ethnic, racial, socioeconomic, geographic, or other groups not broadly participating in a school’s AP program. We believe that readiness for AP is attainable, and that educators can expand readiness by opening access to Pre-AP course work. We commit to support educators and communities in their efforts to make AP courses widely available, advancing students in their plans for college and careers.

## Offering AP Courses: The AP Course Audit

The AP Program unequivocally supports the principle that each school implements its own curriculum that will enable students to develop the content understandings and skills described in the course framework.

While the unit sequence represented in this publication is optional, the AP Program does have a short list of curricular and resource requirements that must be fulfilled before a school can label a course “Advanced Placement” or “AP.” Schools wishing to offer AP courses must participate in the AP Course Audit, a process through which AP teachers’ course materials are reviewed by college faculty. The AP Course Audit was created to provide teachers and administrators with clear guidelines on curricular and resource requirements for AP courses and to help colleges and universities validate courses marked “AP” on students’ transcripts. This process ensures that AP teachers’ courses meet or exceed the curricular and resource expectations that college and secondary school faculty have established for college-level courses.

The AP Course Audit form is submitted by the AP teacher and the school principal (or designated administrator) to confirm awareness and understanding of the curricular and resource requirements. A syllabus or course outline, detailing how course requirements are met, is submitted by the AP teacher for review by college faculty.

Please visit the [AP Course Audit](#) website for more information to support the preparation and submission of materials for the AP Course Audit.

## How the AP Program Is Developed

The scope of content for an AP course and exam is derived from an analysis of hundreds of syllabi and course offerings of colleges and universities. Using this research and data, a committee of college faculty and expert AP teachers work within the scope of the corresponding college course to articulate what students should know and be able to do upon the completion of the AP course. The resulting course framework is the heart of this course and exam description and serves as a blueprint of the content and skills that can appear on an AP Exam.

The AP Test Development Committees are responsible for developing each AP Exam, ensuring the exam questions are aligned to the course framework. The AP Exam development process is a multiyear endeavor; all AP Exams undergo extensive review, revision, piloting, and analysis to ensure that questions are accurate, fair, and valid, and that there is an appropriate spread of difficulty across the questions.

Committee members are selected to represent a variety of perspectives and institutions (public and private, small and large schools and colleges), and a range of gender, racial/ethnic, and regional groups. A list of contributing institutions for each subject is available on [AP Central](#)<sup>®</sup>.

Throughout AP course and exam development, College Board gathers feedback from various stakeholders in both secondary schools and higher education institutions. This feedback is carefully considered to ensure that AP courses and exams are able to provide students with a college-level learning experience and the opportunity to demonstrate their qualifications for advanced placement and/or college credit.

## How AP Exams Are Scored

The exam scoring process, like the course and exam development process, relies on the expertise of both AP teachers and college faculty. While multiple-choice questions are scored by machine, the free-response questions and

through-course performance assessments, as applicable, are scored by thousands of college faculty and expert AP teachers. Most are scored at the annual AP Reading, while a small portion is scored online. All AP Readers are thoroughly trained, and their work is monitored throughout the Reading for fairness and consistency. In each subject, a highly respected college faculty member serves as Chief Faculty Consultant and, with the help of AP Readers in leadership positions, maintains the accuracy of the scoring standards. Scores on the free-response questions and performance assessments are weighted and combined with the results of the computer-scored multiple-choice questions, and this raw score is converted into a composite AP score on a 1–5 scale.

AP Exams are **not** norm referenced or graded on a curve. Instead, they are criterion referenced, which means that every student who meets the criteria for an AP score of 2, 3, 4, or 5 will receive that score, no matter how many students that is. The criteria for the number of points students must earn on the AP Exam to receive scores of 3, 4, or 5—the scores that research consistently validates for credit and placement purposes—include:

- The number of points successful college students earn when their professors administer AP Exam questions to them
- Performance that researchers have found to be predictive of an AP student succeeding when placed into a subsequent higher-level college course
- The number of points college faculty indicate, after reviewing each AP question, that they expect is necessary to achieve each AP grade level

## Using and Interpreting AP Scores

The extensive work done by college faculty and AP teachers in the development of the course and exam and throughout the scoring process ensures that AP Exam scores accurately represent students' achievement in the equivalent college course. Frequent and regular research studies establish the validity of AP scores as follows:

AP Score	Credit Recommendation	College Grade Equivalent
5	Extremely well qualified	A
4	Well qualified	A-, B+, B
3	Qualified	B-, C+, C
2	Possibly qualified	n/a
1	No recommendation	n/a

While colleges and universities are responsible for setting their own credit and placement policies, most private colleges and universities award credit and/or advanced placement for AP scores of 3 or higher. Additionally, most states in the U.S. have adopted statewide credit policies that ensure college credit for scores of 3 or higher at public colleges and universities. To confirm a specific college's AP credit/placement policy, use the search engine available on the [AP Credit Policy Search](#) page.

### BECOMING AN AP READER

Each June, thousands of AP teachers and college faculty members from around the world gather for seven days in multiple locations to evaluate and score the free-response sections of the AP Exams. Ninety-eight percent of surveyed educators who took part in the AP Reading say it was a positive experience.

There are many reasons to consider becoming an AP Reader, including opportunities to:

- **Bring positive changes to the classroom:** Surveys show that the vast majority of returning AP Readers—both high school and college educators—make improvements to the way they teach or score because of their experience at the AP Reading.
- **Gain in-depth understanding of AP Exam and AP scoring standards:** AP Readers gain exposure to the quality and depth of the responses from the entire pool of AP Exam takers and thus are better able to assess their students' work in the classroom.
- **Receive compensation:** AP Readers are compensated for their work during the Reading. Expenses, lodging, and meals are covered for Readers who travel.
- **Score from home:** AP Readers have online distributed scoring opportunities for certain subjects. Check the [AP Reader](#) site for details.
- **Earn Continuing Education Units (CEUs):** AP Readers earn professional development hours and CEUs that can be applied to PD requirements by states, districts, and schools.

#### HOW TO APPLY

Visit the [Become an AP Reader](#) site for eligibility requirements and to start the application process.

# AP Resources and Supports

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By completing a simple activation process at the start of the school year, teachers and students receive access to a robust set of classroom resources.

## AP Classroom

**AP Classroom** is a dedicated online platform designed to support teachers and students throughout their AP experience. The platform provides a variety of powerful resources and tools to provide yearlong support to teachers, offering opportunities to give and get meaningful feedback on student progress.

## UNIT GUIDES

Appearing in this publication and on AP Classroom, these planning guides outline all required course content and skills, organized into commonly taught units. Each unit guide suggests a sequence and pacing of content, scaffolds skill instruction across units, organizes content into topics, and provides tips on taking the AP Exam.

## PROGRESS CHECKS

Formative AP questions for every unit provide feedback to students on the areas where they need to focus. Available online, Progress Checks measure knowledge and skills through multiple-choice questions with rationales to explain correct and incorrect answers, and free-response questions with scoring information.

## REPORTS

Reports provides teachers with a one-stop shop for student results on all assignment types, including Progress Checks. Teachers can view class trends and see where students struggle with content and skills that will be assessed on the AP Exam. Students can view their own progress over time to improve their performance before the AP Exam.

## QUESTION BANK

The Question Bank is a searchable library of all AP questions that teachers use to build custom practice for their students. Teachers can create and assign assessments with formative topic questions or questions from practice or released AP Exams.

### Class Section Setup and Enrollment

- Teachers and students sign into or create their College Board accounts.
- Teachers confirm that they have added the course they teach to their AP Course Audit account and have had it approved by their school's administrator.
- Teachers or AP coordinators, depending on who the school has decided is responsible, set up class sections so students can access AP resources and have exams ordered on their behalf.
- Students join class sections with a join code provided by their teacher or AP coordinator.
- Students will be asked for additional information upon joining their first class section.

# Instructional Model

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Integrating AP resources throughout the course can help students develop skills and conceptual understandings. The instructional model outlined below shows possible ways to incorporate AP resources into the classroom.

## Plan

Teachers may consider the following approaches as they plan their instruction before teaching each unit.

- Review the overview at the start of each **unit guide** to identify essential questions, conceptual understandings, and skills for each unit.
- Use the **Unit at a Glance** table to identify related topics that build toward a common understanding, and then plan appropriate pacing for students.

## Teach

When teaching, supporting resources could be used to build students' conceptual understanding and mastery of skills.

- Use the topic pages in the **unit guides** to identify the required content.
- Integrate the content with a skill, considering any appropriate scaffolding.
- Use the available resources, including **AP Videos**, on the topic pages to bring a variety of assets into the classroom.

## Assess

Teachers can measure student understanding of the content and skills covered in the unit and provide actionable feedback to students.

- While teaching each topic, use **AP Classroom** to assign students **topic questions** as a way to continuously check student understanding and provide just-in-time feedback.
- At the end of each unit, use **AP Classroom** to assign students **progress checks**, as homework or an in-class task.
- Provide question-level feedback to students through answer rationales; provide formative feedback using **Reports**.
- Create additional practice opportunities using the **Question Bank** and assign them through **AP Classroom**.

# About the AP Statistics Course

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The AP Statistics course introduces students to the major concepts and tools for formulating questions, collecting and analyzing data, and interpreting results from data. The practices and skills for the course have been aligned to help students understand the statistical problem-solving process based on the American Statistical Association (ASA) recommendations. The content, skills, and assessments in the AP Statistics course focus on exploring data, sampling and experimentation, probability and simulation, and statistical inference. Students use technology, investigations, problem solving, and writing as they build conceptual understanding.

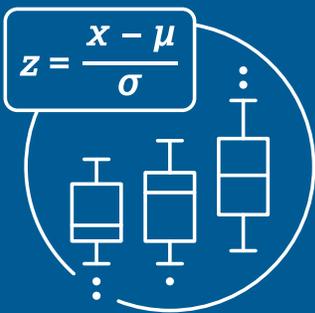
## College Course Equivalent

The AP Statistics course is equivalent to a one-semester, introductory, non-calculus-based college course in statistics.

## Prerequisites

The AP Statistics course is an excellent option for any secondary school student who has successfully completed a first-year algebra course.

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AP STATISTICS

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# Course Framework

# Introduction

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The AP Statistics framework is aligned with content used in college-level courses and skills that follow the statistical problem-solving process identified in the Guidelines for Assessment and Instruction in Statistics Education (GAISE II) from the American Statistical Association (ASA). This framework defines content students must know and skills students must master to learn and retain an understanding of statistics they can apply in academic and everyday endeavors. The framework focuses on providing students with an experience that supports the learning of introductory statistics content and skills.

The framework is organized in a logical sequence, based on teacher input and commonly used textbooks. The sequence represented in the framework demonstrates one reasonable learning pathway for the course, among many. Teachers may adjust the suggested sequencing of units or topics, although they will want to carefully consider how to account for such changes as they access course resources for planning, instruction, and assessment.

# Statistical Practices

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The statistical practices and skills describe what a student should be able to do while exploring course concepts. The table that follows presents the practices, along with their corresponding skills, that students should develop during the AP Statistics course. These skills form the basis of the tasks on the AP Statistics Exam.

The unit guides that follow embed and spiral these skills throughout the course, providing teachers with one way to integrate the skills into the course content, with sufficient repetition to prepare students to apply those skills when taking the AP Statistics Exam. Because the course skills are aligned to specific learning objectives, AP Exam questions will also reflect this pairing.

# Statistical Practices

Practice 1	Practice 2	Practice 3	Practice 4
<b>Formulate Questions</b> 1 Determine an investigative question for a statistical study.	<b>Collect Data</b> 2 Identify and justify methods for collecting data and conducting statistical inference.	<b>Analyze Data</b> 3 Construct representations of data and calculate numerical statistical outputs.	<b>Interpret Results</b> 4 Interpret results and justify conclusions and methods.

## SKILLS

**1.A** Determine a valid investigative question that requires a statistical investigation.

**2.A** Identify information to answer a question or solve a problem.

**2.B** Justify an appropriate method for ethically gathering and representing data.

**2.C** Identify appropriate statistical inference methods.

**2.D** Identify types of errors and relationships among components in statistical inference methods.

**2.E** Identify the null and alternative hypotheses.

**3.A** Construct tabular and graphical representations of data and distributions.

**3.B** Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

**3.C** Calculate and estimate expected counts, percentages, probabilities, and intervals.

**3.D** Calculate means, standard deviations, and parameters for probability distributions.

**3.E** Calculate appropriate statistical inference method results.

**4.A** Describe and compare tabular and graphical representations of data, as well as summary statistics.

**4.B** Justify a claim based on statistical calculations and results.

**4.C** Describe distributions and compare relative positions of points within a distribution.

**4.D** Interpret statistical calculations and results to assess meaning or a claim.

**4.E** Justify the use of a chosen statistical inference method by verifying conditions.

**4.F** Interpret results of statistical inference methods.

**4.G** Justify a claim based on statistical inference method results.

# Course Content

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This course framework provides a clear and detailed description of the course requirements necessary for student success. The framework specifies what students must know, be able to do, and understand, with a focus on ideas that encompass principles and processes in the discipline of statistics. The framework also encourages instruction that prepares students for advanced coursework in statistics or other fields using statistical reasoning and for active, informed engagement with a world of data to be interpreted appropriately and applied wisely to make informed decisions.

## UNITS

The AP Statistics content is subdivided into five units. Pacing recommendations in the Course at a Glance and at the unit level provide suggestions for how to teach the required course content and administer the Progress Checks. The suggested class periods are based on a schedule in which class meets five days a week for 45 minutes each day, for a full school year. While these recommendations have been made to aid planning, teachers should adjust pacing based on the needs of their students, alternate schedules (e.g., block scheduling), or their school's academic calendar.

## TOPICS

Each unit is broken down into teachable segments called topics. The topic pages (starting on p. 30) contain all required content for each topic. Although most topics can be taught in two or three class periods, teachers should pace the course to suit the needs of their students and school.

## Exam Weighting for the Multiple-Choice Section of the AP Exam

Units	Exam Weighting
<b>Unit 1:</b> Exploring One-Variable Data and Collecting Data	20–30%
<b>Unit 2:</b> Probability, Random Variables, and Probability Distributions	15–25%
<b>Unit 3:</b> Inference for Categorical Data: Proportions	15–25%
<b>Unit 4:</b> Inference for Quantitative Data: Means	10–20%
<b>Unit 5:</b> Regression Analysis	10–20%

# Course at a Glance

## Plan

The Course at a Glance provides a useful visual organization of the AP Statistics curricular components, including:

- Sequence of units, along with approximate weighting and suggested pacing. Please note, pacing is based on 45-minute class periods meeting five days each week for a full academic year.
- Progression of topics within each unit
- Statistical practices across units

## Teach

### STATISTICAL PRACTICES

Statistical practices spiral throughout the course.

- |                              |                            |
|------------------------------|----------------------------|
| <b>1</b> Formulate Questions | <b>3</b> Analyze Data      |
| <b>2</b> Collect Data        | <b>4</b> Interpret Results |

## Required Course Content

Each topic contains required Learning Objectives and Essential Knowledge statements that form the basis of the assessment on the AP Exam.

## Assess

Assign the Progress Checks—either as homework or in class—for each unit. Each Progress Check contains formative multiple-choice and free-response questions. The feedback from each Progress Check shows students the areas where they need to focus.

## UNIT 1

### Exploring One-Variable Data and Collecting Data

~26 Class Periods | 20–30% AP Exam Weighting

Statistical Practices	Topics
<b>1</b> <b>2</b> *	<b>1.1</b> Introducing Statistics: What Can We Learn from Data?
<b>2</b>	<b>1.2</b> Variables
<b>3</b> <b>4</b>	<b>1.3</b> Tabular Representation and Summary Statistics for One Categorical Variable
<b>3</b> <b>4</b>	<b>1.4</b> Graphical Representations for One Categorical Variable
<b>3</b>	<b>1.5</b> Graphical Representations for One Quantitative Variable
<b>4</b>	<b>1.6</b> Descriptions for One Quantitative Variable Distributions
<b>3</b> <b>4</b>	<b>1.7</b> Summary Statistics for One Quantitative Variable
<b>3</b> <b>4</b>	<b>1.8</b> Graphical Representations of Summary Statistics for One Quantitative Variable
<b>3</b> <b>4</b>	<b>1.9</b> Comparisons of the Distributions for One Quantitative Variable
<b>1</b> <b>2</b>	<b>1.10</b> The Investigative Question Revisited and Data Collection
<b>2</b>	<b>1.11</b> Random Sampling
<b>2</b>	<b>1.12</b> Potential Problems with Sampling
<b>2</b>	<b>1.13</b> Experimental Design

### Progress Check 1

**Multiple-choice: ~44 questions**

**Free-response: 3 questions**

- Question 1: Multi-Focus on Practices 1 and 2
- Question 2: Multi-Focus on Practices 3 and 4
- Question 2: Multi-Focus on Practices 3 and 4

## UNIT 2

### Probability, Random Variables, and Probability Distributions

~24 Class Periods | 15–25% AP Exam Weighting

Statistical Practices	Topics
<b>4</b>	<b>2.1</b> Tabular and Graphical Representations for the Distributions of Two Categorical Variables
<b>3</b> <b>4</b>	<b>2.2</b> Summary Statistics for Two Categorical Variables
<b>3</b>	<b>2.3</b> Estimating Probabilities Using Simulation
<b>3</b>	<b>2.4</b> Introduction to Probability
<b>4</b>	<b>2.5</b> Mutually Exclusive Events
<b>3</b>	<b>2.6</b> Conditional Probability
<b>3</b>	<b>2.7</b> Independent Events and Unions of Events
<b>3</b>	<b>2.8</b> Introduction to Random Variables and Probability Distributions
<b>3</b> <b>4</b>	<b>2.9</b> Parameters of Random Variables
<b>3</b> <b>4</b>	<b>2.10</b> The Binomial Distribution
<b>3</b> <b>4</b>	<b>2.11</b> The Normal Distribution
<b>4</b>	<b>2.12</b> Sampling Distributions and the Central Limit Theorem

### Progress Check 2

**Multiple-choice: ~42 questions**

**Free-response: 2 questions**

- Question 2: Multi-Focus on Practices 3 and 4
- Question 2: Multi-Focus on Practices 3 and 4

\* **1**  
**2** indicates that Statistical Practices 1 and 2 are covered by Topic 1.1.

# COURSE AT A GLANCE (cont'd)

## UNIT 3

### Inference for Categorical Data: Proportions

~30 Class Periods

15–25% AP Exam Weighting

Statistical Practices	Topics	Statistical Practices	Topics
	3.1 Estimators		3.13 Carrying Out a Test for the Difference Between Two Population Proportions
	3.2 Sampling Distributions for Sample Proportions		3.14 Setting Up a Chi-Square Test for Homogeneity or Independence
	3.3 Constructing a Confidence Interval for a Population Proportion		3.15 Carrying Out a Chi-Square Test for Homogeneity or Independence
	3.4 Justifying a Claim Based on a Confidence Interval for a Population Proportion		
	3.5 Setting Up a Test for a Population Proportion		
	3.6 $p$ -Values		
	3.7 Carrying Out a Test for a Population Proportion		
	3.8 Potential Errors When Performing Tests		
	3.9 Sampling Distributions for the Difference Between Sample Proportions		
	3.10 Constructing a Confidence Interval for the Difference Between Two Population Proportions		
	3.11 Justifying a Claim Based on a Confidence Interval for the Difference Between Two Population Proportions		
	3.12 Setting Up a Test for the Difference Between Two Population Proportions		

#### Progress Check 3

**Multiple-choice:** ~65 questions

**Free-response:** 3 questions

- Question 3: Inference
- Question 3: Inference
- Question 3: Inference

# COURSE AT A GLANCE (cont'd)



**UNIT 4**

## Inference for Quantitative Data: Means

**~18** Class Periods

**10–20%** AP Exam Weighting

Statistical Practices	Topics
<div style="display: flex; align-items: center;"> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.1 Sampling Distributions for Sample Means
<div style="display: flex; align-items: center;"> <div style="background-color: #000080; color: white; padding: 2px 5px; margin-right: 5px;">2</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.2 Constructing a Confidence Interval for a Population Mean or Population Mean Difference
<div style="display: flex; align-items: center;"> <div style="background-color: #000080; color: white; padding: 2px 5px; margin-right: 5px;">2</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.3 Justifying a Claim Based on a Confidence Interval for a Population Mean or Population Mean Difference
<div style="display: flex; align-items: center;"> <div style="background-color: #000080; color: white; padding: 2px 5px; margin-right: 5px;">2</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.4 Setting Up a Test for a Population Mean or Population Mean Difference
<div style="display: flex; align-items: center;"> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.5 Carrying Out a Test for a Population Mean or Population Mean Difference
<div style="display: flex; align-items: center;"> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.6 Sampling Distributions for the Difference Between Two Sample Means
<div style="display: flex; align-items: center;"> <div style="background-color: #000080; color: white; padding: 2px 5px; margin-right: 5px;">2</div> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.7 Constructing a Confidence Interval for the Difference Between Two Population Means
<div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div>	4.8 Justifying a Claim Based on a Confidence Interval for the Difference Between Two Population Means
<div style="display: flex; align-items: center;"> <div style="background-color: #000080; color: white; padding: 2px 5px; margin-right: 5px;">2</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.9 Setting Up a Test for the Difference Between Two Population Means
<div style="display: flex; align-items: center;"> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	4.10 Carrying Out a Test for the Difference Between Two Population Means

### Progress Check 4

**Multiple-choice: ~49 questions**

**Free-response: 3 questions**

- Question 3: Inference
- Question 3: Inference
- Question 3: Inference



**UNIT 5**

## Regression Analysis

**~9** Class Periods

**10–20%** AP Exam Weighting

Statistical Practices	Topics
<div style="display: flex; align-items: center;"> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	5.1 Graphical Representations Between Two Quantitative Variables
<div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div>	5.2 Correlation
<div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div>	5.3 Linear Regression Models
<div style="display: flex; align-items: center;"> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	5.4 Residuals
<div style="display: flex; align-items: center;"> <div style="background-color: #8B4513; color: white; padding: 2px 5px; margin-right: 5px;">3</div> <div style="background-color: #FFD700; padding: 2px 5px; margin-right: 5px;">4</div> </div>	5.5 Least-Squares Regression

### Progress Check 5

**Multiple-choice: ~19 questions**

**Free-response: 2 questions**

- Question 4: Multi-Focus on Practices 2, 3, and 4
- Question 4: Multi-Focus on Practices 2, 3, and 4

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## AP STATISTICS

# Unit Guides

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### Introduction

Designed with extensive input from the community of AP Statistics educators, the unit guides offer teachers helpful guidance in building students' skills and knowledge. The suggested sequence was identified through a thorough analysis of the syllabi of highly effective AP teachers and the organization of typical college textbooks.

This unit structure respects new AP teachers' time by providing one possible sequence they can adopt and modify rather than having to build from scratch. An additional benefit is that these units enable the AP Program to provide interested teachers with formative assessments—the Progress Checks—that they can assign their students at the end of each unit to gauge progress toward success on the AP Exam. However, experienced AP teachers who are satisfied with their current course organization and exam results should feel no pressure to adopt these units, which comprise an optional sequence for this course.

# Using the Unit Guides

UNIT  
4

**10–20%** AP EXAM WEIGHTING  
**~18** CLASS PERIODS

## Inference for Quantitative Data: Means

**ESSENTIAL QUESTIONS**

- How can we compare the average salaries of different professions across different regions of a country?
- How can we evaluate the effectiveness of a new medicine in lowering cholesterol for older adults?
- How can a pre-test and a post-test be used to evaluate student learning?

**Developing Understanding**

In this unit, students analyze quantitative data to make inferences about population means. Students learn that  $t$ - and  $F$ -tests are used for inference with means when the population standard deviation,  $\sigma$ , is unknown. Using  $s$  for  $\sigma$  in the formula for  $z$  gives a slightly different value.  $t$ . The distribution of  $t$  depends on sample size and has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a  $t$ -distribution tend to be further from the mean than for a normal distribution. Students discover how and why conditions for inference with proportions and means are similar and different.

**Building Statistical Practices**

Unit 4 focuses on means, which have similarities to the conditions and procedures for proportions. Because students sometimes confuse  $t$ -tests with  $z$ -tests, teachers might consider carving out time to review the underlying rationales each time conditions are discussed. These reviews help students develop understanding through repeated practice in new situations. Teachers can encourage students to be mindful of notation and reinforce that inference testing requires careful selection of a procedure based on specific conditions for a given problem. Common student errors include mislabeling conditions (e.g., incorrectly associating the 10% condition with the randomization condition), relying on vague references to the normal distribution, or applying the sample data condition incorrectly. The null and alternative hypotheses must be clearly stated in terms of population parameters, not sample statistics. A formal decision compares the  $p$ -value to the level of significance. Students will benefit from having opportunities to practice providing a numerical reference to support their claim (e.g., we reject the null hypothesis because  $p$ -value  $\leq 0.05$ ) and interpret findings within the context of the question.

**Preparing for the AP Exam**

These concepts will be evaluated using individual multiple-choice questions and parts of free-response questions. Students are expected to recognize that free-response questions asking whether data provide convincing evidence of some findings are asking for a significance test, not just a descriptive analysis. When using statistical inference for hypothesis tests, students need to identify the correct parameter and hypotheses, identify an appropriate test procedure and check conditions, calculate a test statistic and  $p$ -value, and provide a conclusion in context, along with a justification based on linkage between the  $p$ -value and the level of significance. Students will benefit from having opportunities to practice inference procedures from Units 3 and 4 so they understand the differences and similarities.

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## UNIT OPENERS

**Developing Understanding** provides an overview that contextualizes and situates the key content of the unit within the scope of the course.

The **essential questions** are thought-provoking questions that motivate students and inspire inquiry.

**Building Statistical Practices** describes specific skills within the practices that are appropriate to focus on in the unit. Certain practices have been noted to indicate areas of emphasis for that unit.

**Preparing for the AP Exam** provides helpful tips and common student misunderstandings identified from prior exam data.

Exploring One-Variable Data and Collecting Data

UNIT  
1

## UNIT AT A GLANCE

Topic	Skills	Class Periods ~26 CLASS PERIODS
1.1 Introducing Statistics: What Can We Learn from Data?	<ul style="list-style-type: none"> <li>1.A Determine a valid investigative question that requires a statistical investigation.</li> <li>1.B Identify information to answer a question or solve a problem.</li> </ul>	2
1.2 Variables	<ul style="list-style-type: none"> <li>1.C Identify information to answer a question or solve a problem.</li> </ul>	1
1.3 Tabular Representation and Summary Statistics for One Categorical Variable	<ul style="list-style-type: none"> <li>1.D Construct tabular and graphical representations of data and distributions.</li> <li>1.E Describe and compare tabular and graphical representations of data, as well as summary statistics.</li> </ul>	2
1.4 Graphical Representations for One Categorical Variable	<ul style="list-style-type: none"> <li>1.F Construct tabular and graphical representations of data and distributions.</li> <li>1.G Describe and compare tabular and graphical representations of data, as well as summary statistics.</li> <li>1.H Justify a claim based on statistical calculations and results.</li> </ul>	2
1.5 Graphical Representations for One Quantitative Variable	<ul style="list-style-type: none"> <li>1.F Construct tabular and graphical representations of data and distributions.</li> </ul>	2
1.6 Descriptions for One Quantitative Variable Distributions	<ul style="list-style-type: none"> <li>1.E Describe and compare tabular and graphical representations of data, as well as summary statistics.</li> <li>1.H Justify a claim based on statistical calculations and results.</li> </ul>	2
1.7 Summary Statistics for One Quantitative Variable	<ul style="list-style-type: none"> <li>1.F Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</li> <li>1.E Describe and compare tabular and graphical representations of data, as well as summary statistics.</li> <li>1.H Justify a claim based on statistical calculations and results.</li> </ul>	2
1.8 Graphical Representations of Summary Statistics for One Quantitative Variable	<ul style="list-style-type: none"> <li>1.F Construct tabular and graphical representations of data and distributions.</li> <li>1.E Describe and compare tabular and graphical representations of data, as well as summary statistics.</li> </ul>	2

(Continued)

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The **Unit at a Glance** table shows the topics and skills.

The skills for each topic show possible ways to link the content in that topic to specific AP Statistics skills. The individual skills have been thoughtfully chosen in a way that scaffolds the skills throughout the course.

# Using the Unit Guides

Exploring One-Variable Data and Collecting Data

UNIT  
1

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit.

Activity	Topics	Sample Activity
1	1.5	<b>Gallery Walk</b> Have students work in groups of four to construct a dotplot, stem-and-leaf plot, histogram, or boxplot for a set of student-generated data (e.g., time in minutes to get to school). After the gallery walk, discuss what information can be seen more easily in each graph (e.g., boxplots can easily show the IQR).
2	1.9	<b>Match Mine</b> Create a blank $3 \times 3$ grid as a gameboard and nine index cards depicting distributions of differing shapes, center, skewness, clustering, etc. Pair up students and place a folder between the partners so they cannot see each other's boards. Distribute a blank grid and a set of cards to each partner. Student A arranges the cards however they choose on their grid and then describes that arrangement so that Student B can attempt to match it on their own board. When the pair thinks they have correctly made all matches, have them compare their arrangements to see how well they did.
3	1.10	<b>Odd One Out</b> After modeling an odd one out example, have students form groups of four and give each student a description of a statistical study. Explain that three of the studies are of the same type (observational or experimental) and one is different. Have students work together in their groups to determine which study is the odd one out and explain why.
4	1.11 1.12	<b>Password-Style Games</b> After completing the lessons on sampling and surveying, use the following nine terms in a password-style game: simple random sample, stratified random sample, cluster random sample, systematic random sample, voluntary response bias, undercoverage bias, nonresponse bias, and response bias. Pair students and ask one student per pair to sit facing the back of the classroom. Project or write the terms on the board one at a time while the front-facing students give clues to their partners, who try to guess the terms. The winner is the pair that guesses the most terms correctly.

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The **Sample Instructional Activities** include optional activities that can help tie together the content and skills for a particular topic.

UNIT  
1

Exploring One-Variable Data and Collecting Data

**TOPIC 1.1**

## Introducing Statistics: What Can We Learn from Data?

**SKILLS**

- 1.A** Determine a valid investigative question that requires a statistical investigation.
- 1.B** Identify information to answer a question or solve a problem.

**Required Course Content**

LEARNING OBJECTIVE	ESSENTIAL KNOWLEDGE
<b>1.1.A</b> Identify components within a statistical study. [Skill 2.A]	<b>1.1.A.1</b> A statistical study is a study in which data are collected from a sample to answer an investigative question about a larger population. <b>1.1.A.2</b> Statistical studies are necessary when the population is too large or it is too difficult to collect data from every item or individual in the population. <b>1.1.A.3</b> A datum (singular form of data) is a piece of information about an item or individual. A collection of data is called a data set. <b>1.1.A.4</b> A population consists of all items or individuals of interest. The population size is represented by the symbol $N$ . <b>1.1.A.5</b> A sample selected for study is a subset of the population from which data are obtained. The number of items in the sample, called the sample size, is represented by the symbol $n$ . <b>1.1.A.6</b> Each component of a statistical study and the resulting calculations can be related to an aspect of the corresponding real-world context from which the components were derived. This identification of a statistical result with the corresponding contextual component is what is meant by "in context."

continued on next page

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## TOPIC PAGES

The **skills** note the course skills that are paired with the learning objectives for that topic.

**Learning objectives** define what a student needs to be able to do with content knowledge to progress through the course.

**Essential knowledge** statements define the required content knowledge associated with each learning objective assessed on the AP Exam.

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## AP STATISTICS

### UNIT 1

# Exploring One-Variable Data and Collecting Data



**20–30%**  
AP EXAM WEIGHTING



**~26**  
CLASS PERIODS



Remember to go to [AP Classroom](#) to assign students the online **Progress Checks** for this unit.

Whether assigned as homework or completed in class, the **Progress Checks** provide each student with immediate feedback related to this unit's topics and skills.

### **Progress Check 1**

**Multiple-choice: ~44 questions**

**Free-response: 3 questions**

- Question 1: Multi-Focus on Practices 1 and 2
- Question 2: Multi-Focus on Practices 3 and 4
- Question 2: Multi-Focus on Practices 3 and 4

# Exploring One-Variable Data and Collecting Data

## ESSENTIAL QUESTIONS

- What kind of question can be answered using data from websites?
- What kind of data is collected about me on social media?
- What methods should I use to collect the data to conduct future inference procedures?



## Developing Understanding

In each unit of the course, students engage with the principles and processes in the discipline of statistics. Starting in Unit 1, students identify components and formulate investigative questions of a statistical study, define and represent categorical and quantitative variables, describe and compare distributions of one-variable data, interpret statistical calculations to assess claims about individual data points or samples, and justify the use of important principles of sampling and experimental design. Students also learn to talk about data in real-world contexts. Variability in data may suggest certain conclusions about the data distribution, but not all variation is meaningful. Statistics allow us to develop shared understandings of uncertainty and variation.

Additionally, students learn that, depending on how data are collected, they may or may not be able to generalize findings or establish evidence of causal relationships. For example, if random selection is not used to obtain a sample from a population, bias may result and statistics from the sample could not be assumed to generalize to the population. For data collected using well-designed experiments, statistically significant differences between or among experimental treatment groups are evidence that the treatments caused the effect.

## Building Statistical Practices

1.A 2.A 2.B 3.A 3.B 4.A 4.B 4.C

Having access to a world of data is meaningless without the ability to organize and analyze that information. Providing students with multiple opportunities to interact with data presented in different formats (e.g., as a table, a graph, or even just a list of values) can help them develop these skills. Students learn to describe the patterns and characteristics they see in the data—including shape, center, variability, and unusual features for a quantitative variable—and then compare the characteristics of two different sets of data. Students also create displays that appropriately represent the data (e.g., using a bar graph for categorical data). Providing explicit feedback on students' responses facilitates their understanding of the level of detail needed. For example, when students describe a distribution of quantitative data, they often provide an acronym such as SOCS (Shape, Outliers/Gaps, Center, Spread) associated with that type of distribution but then struggle to explain all the elements in the acronym. In particular, students often neglect to discuss unusual features such as gaps or outliers. Teachers should reinforce the need for students to address these elements in their descriptions and convey to students that all data has context (e.g., the variable of interest, including any units of measurement).

Statisticians are adept at determining what the question asks. Identifying the task in the given prompt and checking that their response addresses that task helps students develop this skill. For example, when asked whether it is appropriate to generalize the results of a given experiment, students learn to provide a clear yes or no decision in their response, along with an explanation supporting that decision. Although students may recognize the need to justify their reasoning, they often struggle to include explicit evidence supporting their claims. For instance,

some students forget to support claims about nonresponse bias with evidence indicating whether the sample result is likely to be too high or too low compared to the population value being estimated. As another example, students need to clearly explain why a particular variable might lead to confounding in a given setting.

### Preparing for the AP Exam

These concepts will be evaluated using individual multiple-choice questions and parts of free-response questions. To help students prepare for the AP Statistics Exam, plan to review the different components of the investigative questions and their importance in statistical studies. Encourage students to carefully read each question and completely answer the question asked. For example, when interpreting representations of quantitative data, students are expected to describe shape, center, and variability, as well as unusual features like outliers. A response focused only on the center, for example, is incomplete. Students also need to provide complete explanations in context for all conclusions made from data. If asked to justify the selection of a particular conclusion over other options, students must include both the reasoning for their choice and rationales for not choosing the others.

Students need to practice using precise language in their writing for the AP Statistics Exam. For example, some students write about random selection when the question refers to random assignment, or vice versa. Precise student answers discuss random selection when generalizing results of a sample to the population and refer to random assignment when explaining experiments or their results. Because random assignment to treatments in an experiment tends to balance the effects of uncontrolled variables across groups, researchers can conclude that statistically significant differences in the response are caused by the effects of the treatments. When justifying a claim that experimental treatments cause such statistically significant differences, a complete student answer cites the random assignment of treatments and does not refer to other irrelevant elements of well-designed experiments. As a final step in the problem-solving process, students can practice connecting responses to the specific context of the question.

## UNIT AT A GLANCE

Topic	Skills	Class Periods ~26 CLASS PERIODS
<b>1.1 Introducing Statistics: What Can We Learn from Data?</b>	<p><b>1.A</b> Determine a valid investigative question that requires a statistical investigation.</p> <p><b>2.A</b> Identify information to answer a question or solve a problem.</p>	2
<b>1.2 Variables</b>	<p><b>2.A</b> Identify information to answer a question or solve a problem.</p>	1
<b>1.3 Tabular Representation and Summary Statistics for One Categorical Variable</b>	<p><b>3.A</b> Construct tabular and graphical representations of data and distributions.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p>	2
<b>1.4 Graphical Representations for One Categorical Variable</b>	<p><b>3.A</b> Construct tabular and graphical representations of data and distributions.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	2
<b>1.5 Graphical Representations for One Quantitative Variable</b>	<p><b>3.A</b> Construct tabular and graphical representations of data and distributions.</p>	2
<b>1.6 Descriptions for One Quantitative Variable Distributions</b>	<p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	2
<b>1.7 Summary Statistics for One Quantitative Variable</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	2
<b>1.8 Graphical Representations of Summary Statistics for One Quantitative Variable</b>	<p><b>3.A</b> Construct tabular and graphical representations of data and distributions.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p>	2

(Continued)

## UNIT AT A GLANCE (cont'd)

Topic	Skills	Class Periods ~26 CLASS PERIODS
<b>1.9 Comparisons of the Distributions for One Quantitative Variable</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p> <p><b>4.C</b> Describe distributions and compare relative positions of points within a distribution.</p>	2
<b>1.10 The Investigative Question Revisited and Data Collection</b>	<p><b>1.A</b> Determine a valid investigative question that requires a statistical investigation.</p> <p><b>2.A</b> Identify information to answer a question or solve a problem.</p> <p><b>2.B</b> Justify an appropriate method for ethically gathering and representing data.</p>	2
<b>1.11 Random Sampling</b>	<p><b>2.A</b> Identify information to answer a question or solve a problem.</p> <p><b>2.B</b> Justify an appropriate method for ethically gathering and representing data.</p>	2
<b>1.12 Potential Problems with Sampling</b>	<p><b>2.A</b> Identify information to answer a question or solve a problem.</p>	2
<b>1.13 Experimental Design</b>	<p><b>2.A</b> Identify information to answer a question or solve a problem.</p> <p><b>2.B</b> Justify an appropriate method for ethically gathering and representing data.</p>	3
<p>Go to <b>AP Classroom</b> to assign the <b>Progress Check</b> for Unit 1. Review the results in class to identify and address any student misunderstandings.</p>		

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit.

Activity	Topics	Sample Activity
1	1.5	<p><b>Gallery Walk</b></p> <p>Have students work in groups of four to construct a dotplot, stem-and-leaf plot, histogram, or boxplot for a set of student-generated data (e.g., time in minutes to get to school). After the gallery walk, discuss what information can be seen more easily in each graph (e.g., boxplots can easily show the IQR).</p>
2	1.9	<p><b>Match Mine</b></p> <p>Create a blank <math>3 \times 3</math> grid as a gameboard and nine index cards depicting distributions of differing shapes, center, skewness, clustering, etc. Pair up students and place a folder between the partners so they cannot see each other's boards. Distribute a blank grid and a set of cards to each partner. Student A arranges the cards however they choose on their grid and then describes that arrangement so that Student B can attempt to match it on their own board. When the pair thinks they have correctly made all matches, have them compare their arrangements to see how well they did.</p>
3	1.10	<p><b>Odd One Out</b></p> <p>After modeling an odd one out example, have students form groups of four and give each student a description of a statistical study. Explain that three of the studies are of the same type (observational or experimental) and one is different. Have students work together in their groups to determine which study is the odd one out and explain why.</p>
4	1.11 1.12	<p><b>Password-Style Games</b></p> <p>After completing the lessons on sampling and surveying, use the following nine terms in a password-style game: simple random sample, stratified random sample, cluster random sample, systematic random sample, bias, voluntary response bias, undercoverage bias, nonresponse bias, and response bias. Pair students and ask one student per pair to sit facing the back of the classroom. Project or write the terms on the board one at a time while the front-facing students give clues to their partners, who try to guess the terms. The winner is the pair that guesses the most terms correctly.</p>

## SKILLS

## 1.A

Determine a valid investigative question that requires a statistical investigation.

## 2.A

Identify information to answer a question or solve a problem.

## TOPIC 1.1

# Introducing Statistics: What Can We Learn from Data?

## Required Course Content

### LEARNING OBJECTIVE

**1.1.A**

Identify components within a statistical study.

[Skill 2.A]

### ESSENTIAL KNOWLEDGE

**1.1.A.1**

A statistical study is a study in which data are collected from a sample to answer an investigative question about a larger population.

**1.1.A.2**

Statistical studies are necessary when the population is too large or it is too difficult to collect data from every item or individual in the population.

**1.1.A.3**

A datum (singular form of data) is a piece of information about an item or individual. A collection of data is called a data set.

**1.1.A.4**

A population consists of all items or individuals of interest. The population size is represented by the symbol  $N$ .

**1.1.A.5**

A sample selected for study is a subset of the population from which data are obtained. The number of items in the sample, called the sample size, is represented by the symbol  $n$ .

**1.1.A.6**

Each component of a statistical study and the resulting calculations can be related to an aspect of the corresponding real-world context from which the components were derived. This identification of a statistical result with the corresponding contextual component is what is meant by “in context.”

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### LEARNING OBJECTIVE

**1.1.B**

Determine an investigative question within a statistical study.

**[Skill 1.A]**

### ESSENTIAL KNOWLEDGE

**1.1.B.1**

An investigative question for a specific study should have a defined purpose and should not be changed based on the data analysis or results.

**1.1.B.2**

An investigative question should be posed so that the required data can be collected and analyzed.

## SKILLS

## 2.A

Identify information to answer a question or solve a problem.

## TOPIC 1.2

# Variables

### Required Course Content

#### LEARNING OBJECTIVE

**1.2.A**

Identify observational units, variables, parameters, and statistics from a statistical study or data set.

[Skill 2.A]

**1.2.B**

Identify types of variables.

[Skill 2.A]

#### ESSENTIAL KNOWLEDGE

**1.2.A.1**

An observational unit is an item or individual from which a datum is collected.

**1.2.A.2**

A variable is a characteristic that may change from one observational unit to another.

**1.2.A.3**

Data collected on numerical and categorical variables measured on observational units, including photographs, sounds, videos, and text, can convey meaningful information.

**1.2.A.4**

A parameter is a numerical attribute or summary of the variable of interest for a population.

**1.2.A.5**

A statistic is a numerical attribute or summary of the variable of interest for a sample. The value of a statistic from a certain sample is often not equal to the unknown value of the population parameter but may provide the basis for making inferences about the population parameter.

**1.2.B.1**

A categorical variable, also called a qualitative variable, takes on values that are category names or group labels.

**1.2.B.2**

A quantitative variable, also called a numerical variable, takes on numerical values for a measured or counted quantity and generally has units of measure.

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### LEARNING OBJECTIVE

**1.2.C**

Identify types of quantitative variables.

**[Skill 2.A]**

### ESSENTIAL KNOWLEDGE

**1.2.C.1**

A discrete quantitative variable can take on a countable number of values. The number of values may be finite or countably infinite, as with the whole numbers.

**1.2.C.2**

A continuous quantitative variable can take on an infinite number of possible values within a given interval. The number of values the variable can take on is measurable but not countable. This variable can take on all possible values between any pair of values.

## SKILLS

## 3.A

Construct tabular and graphical representations of data and distributions.

## 4.A

Describe and compare tabular and graphical representations of data, as well as summary statistics.

## TOPIC 1.3

# Tabular Representation and Summary Statistics for One Categorical Variable

## Required Course Content

### LEARNING OBJECTIVE

**1.3.A**

Construct categorical one-variable tabular representations.

[Skill 3.A]

**1.3.B**

Describe categorical one-variable tabular representations with summary statistics.

[Skill 4.A]

### ESSENTIAL KNOWLEDGE

**1.3.A.1**

A frequency table shows the number of observational units in each category of a categorical variable.

**1.3.A.2**

A relative frequency table shows the proportion of observational units in each category of a categorical variable.

**1.3.B.1**

Percentages, relative frequencies, and ratios all provide the same information as proportions.

**1.3.B.2**

Counts and relative frequencies of categorical variables reveal information that can be used to justify claims about the variables in context.

## TOPIC 1.4

# Graphical Representations for One Categorical Variable

## Required Course Content

### LEARNING OBJECTIVE

**1.4.A**

Construct categorical one-variable graphical representations.

[Skill 3.A]

**1.4.B**

Justify a claim using categorical one-variable graphical representations.

[Skill 4.B]

**1.4.C**

Compare multiple categorical one-variable tabular and graphical representations.

[Skill 4.A]

### ESSENTIAL KNOWLEDGE

**1.4.A.1**

Bar charts, also called bar graphs, display frequencies (counts) or relative frequencies (proportions) for the categories of a single categorical variable. Each bar on a bar chart represents a category of the categorical variable of interest. The height or length of each bar corresponds to the frequency or relative frequency of the observational units in each category.

**1.4.A.2**

Pie charts are used to display frequencies (counts) or relative frequencies (proportions) for categorical data. Each slice on a pie chart represents a category of the categorical variable of interest. The area of each slice, as a fraction of the total area, corresponds to the relative frequency of observational units falling within each category. The sum of the slices' areas together will equal 1, or 100% of the total area.

**1.4.B.1**

Graphical representations of a categorical variable reveal information that can be used to justify claims about the variable in context.

**1.4.C.1**

Frequency and relative frequency tables, bar charts, and pie charts can be used to compare two or more data sets in terms of the same categorical variable.

**SKILLS****3.A**

Construct tabular and graphical representations of data and distributions.

**4.A**

Describe and compare tabular and graphical representations of data, as well as summary statistics.

**4.B**

Justify a claim based on statistical calculations and results.

## SKILLS

## 3.A

Construct tabular and graphical representations of data and distributions.

## TOPIC 1.5

# Graphical Representations for One Quantitative Variable

## Required Course Content

### LEARNING OBJECTIVE

**1.5.A**

Construct quantitative one-variable graphical representations.

[Skill 3.A]

### ESSENTIAL KNOWLEDGE

**1.5.A.1**

Histograms, stem-and-leaf plots, and dotplots provide a visual representation of the distribution of the values of a quantitative variable. These graphs show the frequency or relative frequency of the quantitative variable values or intervals of values and maintain the natural ordering, smallest to largest, of the quantitative variable.

**1.5.A.2**

A histogram places the observed values of the quantitative variable into ordered intervals, or bins, along the horizontal axis. Each bar represents an interval or bin, and the height of each bar shows the frequency or relative frequency of the observations within that interval. Altering the interval widths, or bin widths, can change the appearance of the histogram. Alternatively, a histogram can be constructed with bins on the vertical axis with bars appearing horizontally.

**1.5.A.3**

A stem-and-leaf plot splits each value of the quantitative variable into two parts: a “stem” (the first digit or digits) and a “leaf” (usually the single digit after the stem digit or digits). Both stems and leaves are ordered from smallest to largest.

**1.5.A.4**

A dotplot represents each value of the quantitative variable by a dot. Each dot is placed above the horizontal or beside the vertical axis corresponding to the value of that observation, with nearly identical values stacked on top of each other.

## TOPIC 1.6

# Descriptions for One Quantitative Variable Distributions

## SKILLS

## 4.A

Describe and compare tabular and graphical representations of data, as well as summary statistics.

## 4.B

Justify a claim based on statistical calculations and results.

## Required Course Content

### LEARNING OBJECTIVE

## 1.6.A

Describe distributions of quantitative one-variable graphical representations.

[Skill 4.A]

### ESSENTIAL KNOWLEDGE

## 1.6.A.1

Descriptions of the distribution of one quantitative variable include shape, center, and variability (spread) as well as any unusual features such as outliers, gaps, or clusters in context.

## 1.6.A.2

The shape of the distribution of one quantitative variable is skewed to the right (positively skewed) if the right tail (toward larger values) is longer than the left. The shape of the distribution is skewed to the left (negatively skewed) if the left tail (toward smaller values) is longer than the right. The shape of the distribution is approximately symmetric if the left half is approximately the mirror image of the right half.

## 1.6.A.3

Distributions of one quantitative variable with one main peak are called unimodal. Distributions with two prominent peaks are called bimodal. A distribution in which each frequency or each relative frequency is approximately the same with no prominent peaks is approximately uniform.

## 1.6.A.4

Outliers for one quantitative variable are data points that are unusually small or large relative to the rest of the data.

## 1.6.A.5

A gap is a region in a distribution between two values in which there are no observed data.

## 1.6.A.6

Clusters are concentrations of values usually separated by gaps.

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**LEARNING OBJECTIVE****1.6.B**

Justify a claim using distributions of quantitative one-variable graphical representations.

**[Skill 4.B]**

**ESSENTIAL KNOWLEDGE****1.6.B.1**

Graphical representations of a quantitative variable may reveal information that can be used to justify claims about the variable in context.

## TOPIC 1.7

# Summary Statistics for One Quantitative Variable

## Required Course Content

### LEARNING OBJECTIVE

**1.7.A**

Calculate measures of center and position for quantitative data.

[Skill 3.B]

### ESSENTIAL KNOWLEDGE

**1.7.A.1**

Two commonly used measures of center in the distribution of a quantitative variable are the mean and median.

**1.7.A.2**

The mean is the sum of all the values divided by the number of values and can be found with and without using technology. For a sample, the

mean is denoted by  $\bar{x}$ :  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ , where  $x_i$

represents the  $i$ th data point in the sample and  $n$  represents the number of data values in the sample.

**1.7.A.3**

The median is the middle value when the data set is ordered from smallest to largest and can be found with and without using technology. One common method for determining the median of a data set with an even number of values is to use the mean of the two middle values. A common method for determining the median of a data set with an odd number of values is to use the value in the middle of all the values.

**1.7.A.4**

In an ordered data set, the smallest value is the minimum value, and the largest value is the maximum value.

## SKILLS

**3.B**

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

**4.A**

Describe and compare tabular and graphical representations of data, as well as summary statistics.

**4.B**

Justify a claim based on statistical calculations and results.

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## LEARNING OBJECTIVE

**1.7.A**

Calculate measures of center and position for quantitative data.

[Skill 3.B]

**1.7.B**

Calculate measures of variability for quantitative data.

[Skill 3.B]

## ESSENTIAL KNOWLEDGE

**1.7.A.5**

The first quartile, denoted by  $Q_1$ , is the median value of the lower half of the ordered data set from the minimum value to the position of the median. Approximately 25% of the values in the data set are less than or equal to  $Q_1$ . The third quartile, denoted by  $Q_3$ , is the median value of the upper half of the ordered data set from the position of the median to the maximum value. Approximately 75% of the values in the data set are less than or equal to  $Q_3$ . The second quartile,  $Q_2$ , is also the median of the data set.  $Q_1$  and  $Q_3$  form the boundaries for the middle 50% of values in an ordered data set.

**1.7.A.6**

The  $p$ th percentile is the value that has  $p\%$  of the data less than or equal to it when the data set is ordered from smallest to largest. The first and third quartiles are the 25th and 75th percentiles, respectively.

**1.7.B.1**

Three commonly used measures of variability (or spread) in the distribution of a quantitative variable are the range, interquartile range, and standard deviation.

**1.7.B.2**

The range is the difference between the maximum data value and the minimum data value.

**1.7.B.3**

The interquartile range (IQR) is the difference between the third and first quartiles:  $Q_3 - Q_1$ .

**1.7.B.4**

The standard deviation is a typical deviation of the data values from their mean and can be found with and without using technology. The sample standard deviation is denoted by  $s$  and

calculated by  $s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$ , where  $x_i$

is the data value,  $\bar{x}$  is the mean, and  $n$  is the number of data values in the sample. The square of the sample standard deviation,  $s^2$ , is called the sample variance.

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**LEARNING OBJECTIVE****1.7.C**

Calculate different units of measurement for summary statistics.

**[Skill 3.B]**

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**1.7.D**

Calculate outliers for quantitative data.

**[Skill 3.B]**

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**1.7.E**

Compare multiple quantitative one-variable summary statistics.

**[Skill 4.A]**

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**1.7.F**

Justify the selection of a summary statistic for describing quantitative data.

**[Skill 4.B]**

**ESSENTIAL KNOWLEDGE****1.7.C.1**

Changing units of measurement affects the values of the calculated statistics.

---

**1.7.D.1**

There are many methods for determining potential outliers. Two methods frequently used are as follows:

**1.7.D.1.i**

An outlier is a value located more than  $1.5 \times \text{IQR}$  above the third quartile or more than  $1.5 \times \text{IQR}$  below the first quartile.

**1.7.D.1.ii**

An outlier is a value located more than 2 standard deviations above, or below, the mean.

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**1.7.E.1**

Summary statistics can be used to compare features of two or more independent samples, including center, variability, shape, and outliers.

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**1.7.F.1**

The median and IQR are considered a resistant (or robust) measure of center and measure of variability, respectively, because outliers do not greatly (if at all) affect their values. Because outliers can affect their values greatly, the mean is considered a nonresistant (or non-robust) measure of center, and the range and standard deviation are considered nonresistant (or non-robust) measures of variability.

**1.7.F.2**

Summary statistics of a quantitative variable may reveal information that can be used to justify claims about the variable in context.

## SKILLS

## 3.A

Construct tabular and graphical representations of data and distributions.

## 4.A

Describe and compare tabular and graphical representations of data, as well as summary statistics.

## TOPIC 1.8

# Graphical Representations of Summary Statistics for One Quantitative Variable

## Required Course Content

### LEARNING OBJECTIVE

## 1.8.A

Construct quantitative one-variable graphical representations of summary statistics.

[Skill 3.A]

## 1.8.B

Describe quantitative one-variable graphical representations of summary statistics based on the relationship of the mean and the median.

[Skill 4.A]

### ESSENTIAL KNOWLEDGE

## 1.8.A.1

A five-number summary is made up of the minimum data value, the first quartile (Q1), the median, the third quartile (Q3), and the maximum data value.

## 1.8.A.2

A boxplot is a graphical representation of the five-number summary (minimum, first quartile, median, third quartile, maximum). The box represents the middle 50% of data, with a line at the median and the ends of the box corresponding to the quartiles. Lines (“whiskers”) that represent 25% of the data extend from the first quartile to the minimum and from the third quartile to the maximum. If there are outliers in the data, the whiskers extend to the most extreme data values that are not outliers, and outliers are usually denoted with an asterisk or other symbol.

## 1.8.B.1

If a distribution is relatively symmetric, then the values of the mean and median are relatively close to each other. If a distribution is skewed right, then the value of the mean is usually larger than the median. If the distribution is skewed left, then the value of the mean is usually smaller than the median.

## TOPIC 1.9

# Comparisons of the Distributions for One Quantitative Variable

## Required Course Content

### LEARNING OBJECTIVE

**1.9.A**

Compare multiple quantitative one-variable graphical representations.

[Skill 4.A]

**1.9.B**

Compare multiple quantitative one-variable graphical representations of summary statistics.

[Skill 4.A]

**1.9.C**

Justify a claim using multiple quantitative one-variable graphical representations.

[Skill 4.B]

**1.9.D**

Calculate z-scores with population parameters.

[Skill 3.B]

### ESSENTIAL KNOWLEDGE

**1.9.A.1**

Graphical representations of a quantitative variable can be used to compare important features between two or more distributions of the same quantitative variable. Histograms, back-to-back stem-and-leaf plots, and dotplots may be used to compare center, variability, shape, outliers, clusters, or gaps in two or more distributions. Boxplots may be used to compare center, variability, outliers, and skewness (or symmetry).

**1.9.B.1**

A comparison of graphical representations for two or more distributions can include any of the numerical summaries (e.g., mean, standard deviation, etc.).

**1.9.C.1**

Multiple quantitative one-variable graphical representations may reveal information that can be used to justify claims about the variable in context.

**1.9.D.1**

A standardized score measures the number of standard deviations a data value falls above or below the mean.

### SKILLS

**3.B**

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

**4.A**

Describe and compare tabular and graphical representations of data, as well as summary statistics.

**4.B**

Justify a claim based on statistical calculations and results.

**4.C**

Describe distributions and compare relative positions of points within a distribution.

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## LEARNING OBJECTIVE

**1.9.D**

Calculate z-scores with population parameters.

[Skill 3.B]

**1.9.E**

Compare z-scores as measures of relative position for distributions.

[Skill 4.C]

## ESSENTIAL KNOWLEDGE

**1.9.D.2**

A z-score is calculated as  $\frac{x_i - \mu}{\sigma}$ , where  $x_i$  is the data value,  $\mu$  is the population mean, and  $\sigma$  is the population standard deviation. A z-score measures how many standard deviations a data value is above (positive z-score) or below (negative z-score) the mean. When the population mean and standard deviation are unknown, the sample mean and standard deviation may be used to determine a z-score.

**1.9.E.1**

z-scores may be used to compare relative positions of individual values within a distribution or between distributions.

## TOPIC 1.10

# The Investigative Question Revisited and Data Collection

## SKILLS

## 1.A

Determine a valid investigative question that requires a statistical investigation.

## 2.A

Identify information to answer a question or solve a problem.

## 2.B

Justify an appropriate method for ethically gathering and representing data.

## Required Course Content

### LEARNING OBJECTIVE

## 1.10.A

Determine the components of an investigative question within a statistical study.

[Skill 1.A]

## 1.10.B

Identify a census.

[Skill 2.A]

### ESSENTIAL KNOWLEDGE

## 1.10.A.1

The first component of an investigative question should guide the data collection process and should be phrased in terms of the variable(s) of interest in the study.

## 1.10.A.2

The second component of an investigative question should guide the data analysis choice.

## 1.10.A.2.i

In the case of a hypothesis test, the investigative question should make clear the parameter and the direction of the alternative hypothesis (i.e., not equal, greater than, less than, association, not independent).

## 1.10.A.2.ii

In the case of a confidence interval, the investigative question should make clear the parameter and the goal of estimation of that parameter within a range of potential values.

## 1.10.A.3

The third component of an investigative question should indicate the type(s) of conclusion(s) applicable from the study. The investigative question should provide the population to which the conclusions will be applicable and, in the case of an experiment that uses random assignment, a cause-and-effect conclusion.

## 1.10.B.1

A census consists of recording information from all items or individuals in a population.

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**LEARNING OBJECTIVE****1.10.C**

Identify an experiment.

**[Skill 2.A]**

**1.10.D**

Identify an observational study.

**[Skill 2.A]**

**ESSENTIAL KNOWLEDGE****1.10.C.1**

An experiment is a study in which a researcher assigns conditions, or treatments, to experimental units to explore an investigative question of interest about the population.

**1.10.C.2**

The experimental unit is the observational unit to which the treatment is assigned. When experimental units consist of people, they are sometimes referred to as subjects or participants.

**1.10.C.3**

An explanatory variable, or factor, is a variable whose different categories, or levels, are imposed on the experimental units. The different categories, or levels, of the explanatory variable are called treatments. When there is more than one explanatory variable, the combinations of the categories, or levels, of the explanatory variables are called treatments.

**1.10.C.4**

A response variable is an outcome measured on each experimental unit after the treatment has been administered.

**1.10.D.1**

An observational study is a study where treatments are not imposed. The researcher records the values of the variables of interest in order to explore an investigative question of interest.

**1.10.D.2**

A prospective study is one in which the observational units of study are selected at a point in time, and data are gathered both at that time and into the future.

**1.10.D.3**

A retrospective study is one in which the observational units of study are selected at a point in time and data from the past are gathered.

**1.10.D.4**

A survey is an observational study in which the data are collected from humans using a standard set of questions.

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**LEARNING OBJECTIVE****1.10.D**

Identify an observational study.

[Skill 2.A]

**1.10.E**

Justify the appropriateness of generalizations for a statistical study.

[Skill 2.B]

**ESSENTIAL KNOWLEDGE****1.10.D.5**

A confounding variable in an observational study provides an alternative explanation for the observed relationship between the explanatory and response variables determined in the study, thereby reducing the possibility of concluding a causal relationship between the explanatory and response variables of interest. To be a confounding variable, a variable must be associated with both the explanatory variable and the response variable.

**1.10.E.1**

A sample is considered random when all observational units in the sample are selected from the population using some type of random mechanism, such as a random number generator.

**1.10.E.2**

When observational units, or experimental units, in a sample are randomly selected from a population, it is appropriate to make generalizations about the entire population of individuals from which the sample was selected.

**1.10.E.3**

A sample is not randomly selected when observational units are deliberately chosen or volunteer themselves to be in the sample.

**1.10.E.4**

When observational units, or experimental units, in a sample are not randomly selected from a population, it is appropriate to make generalizations only about a population of individuals that are similar to those used in the study.

## SKILLS

## 2.A

Identify information to answer a question or solve a problem.

## 2.B

Justify an appropriate method for ethically gathering and representing data.

## TOPIC 1.11

# Random Sampling

## Required Course Content

### LEARNING OBJECTIVE

**1.11.A**

Identify a sampling method given a description of a study.

[Skill 2.A]

### ESSENTIAL KNOWLEDGE

**1.11.A.1**

Sampling without replacement is a sampling strategy in which an observational unit from a population can be selected only once. The observational unit is not returned to the population before subsequent selections of observational units are made, so there is no chance that the observational unit can be selected again.

**1.11.A.2**

Sampling with replacement is a sampling strategy in which an observational unit from the population can be selected more than once. The observational unit is returned to the population before subsequent selections of observational units are made, so it is possible that the observational unit could be selected again.

**1.11.A.3**

In a simple random sample (SRS) of size  $n$ , every sample of the size  $n$  has the same chance of being selected. This method is the basis for many types of sampling mechanisms. There are several procedures to obtain a simple random sample; for example, using a random number generator or randomly selecting numbered slips of paper.

**1.11.A.4**

A stratified random sample involves the division of all individuals in a population into non-overlapping groups, called strata, based on one or more shared attributes or characteristics (homogeneous grouping). Within each stratum a simple random sample is selected, and the selected individuals are combined to form one sample.

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## LEARNING OBJECTIVE

### 1.11.A

Identify a sampling method given a description of a study.

[Skill 2.A]

### 1.11.B

Justify the appropriateness of a sampling method.

[Skill 2.B]

## ESSENTIAL KNOWLEDGE

### 1.11.A.5

A cluster random sample involves the division of a population into smaller groups, called clusters. Ideally, each cluster mirrors the heterogeneity of the population, with clusters similar to one another. A simple random sample of clusters is selected from the population to form the sample of clusters. Data are collected from all observational units in each of the selected clusters.

### 1.11.A.6

A systematic random sample is a method in which sample members from a population are selected according to a random starting point and a fixed, periodic interval between successive sampling units.

### 1.11.B.1

Each random sampling method has different characteristics that make it more appropriate for sampling populations depending on the question being investigated.

## SKILLS

## 2.A

Identify information to answer a question or solve a problem.

## TOPIC 1.12

# Potential Problems with Sampling

## Required Course Content

### LEARNING OBJECTIVE

**1.12.A**

Identify potential sources of bias in sampling methods.

[Skill 2.A]

### ESSENTIAL KNOWLEDGE

**1.12.A.1**

Bias in a sampling method is a systematic error in the sampling procedure that results in a statistic being consistently larger or consistently smaller than the parameter the statistic is used to estimate.

**1.12.A.2**

Voluntary response bias is a bias that may occur when a sample consists entirely of volunteers.

**1.12.A.3**

Undercoverage bias may occur when the sampling method fails to include part of the population or a part of the population is less likely to be selected based on the sampling method.

**1.12.A.4**

Nonresponse bias may occur because of a failure to obtain responses from some individuals chosen to be sampled. The respondents and nonrespondents could differ significantly in ways that are important for the study.

**1.12.A.5**

Response bias may occur when responses to a survey or measurements of observational units tend to differ from the “true” value in one direction. Examples include questions that are confusing or leading (question wording bias) or self-reported responses.

**1.12.A.6**

Nonrandom sampling methods (e.g., samples chosen by convenience or voluntary response) introduce potential bias because they do not use random chance to select the individuals.

## TOPIC 1.13

# Experimental Design

## SKILLS

**2.A**

Identify information to answer a question or solve a problem.

**2.B**

Justify an appropriate method for ethically gathering and representing data.

## Required Course Content

### LEARNING OBJECTIVE

**1.13.A**

Identify elements of a well-designed experiment.

[Skill 2.A]

### ESSENTIAL KNOWLEDGE

**1.13.A.1**

A well-designed experiment should include the following:

**1.13.A.1.i**

Comparisons of at least two treatment groups, one of which could be a control group

**1.13.A.1.ii**

Random assignment of treatments to experimental units

**1.13.A.1.iii**

Replication

**1.13.A.1.iv**

Direct control of potential extraneous sources of variation in the response

**1.13.A.2**

A control group is a collection of experimental units that are created for comparison. A control group may be given a treatment different from the treatment of interest to determine if the treatment of interest has an effect (e.g., a treatment with an inactive substance, a placebo, may be given).

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## LEARNING OBJECTIVE

## 1.13.A

Identify elements of a well-designed experiment.

[Skill 2.A]

## ESSENTIAL KNOWLEDGE

## 1.13.A.3

The placebo effect is the difference between the average response to a placebo and the average response to no treatment.

## 1.13.A.4

In a single-blind, also called single-masked, experiment, participants do not know which treatment they are receiving, but members of the research team who interact with them know which treatment each participant is receiving, or vice versa.

## 1.13.A.5

In a double-blind, also called double-masked, experiment, neither the participants nor the members of the research team who interact with them know which treatment each participant is receiving.

## 1.13.A.6

An extraneous source of variation, also referred to as an extraneous variable, is a variable that is known (or believed) to affect the response but is not an explanatory variable being studied.

## 1.13.A.7

The purpose of random assignment is to create treatment groups that are as similar as possible with respect to extraneous sources of variation. If random assignment is successful, the respective distributions of each extraneous variable will be approximately the same for all the treatment groups.

## 1.13.A.8

A confounding variable in an experiment is a variable that is related to the explanatory variable in such a way that it is difficult to determine which variable, explanatory or confounding, is influencing the change in the response variable. However, in a well-designed experiment, the potential for confounding variables is reduced.

## 1.13.A.9

Replication within an experiment means more than one experimental unit is assigned to each treatment.

## 1.13.A.10

Direct control in an experiment means keeping the settings of certain potential extraneous sources of variation in the response variable the same from experimental unit to experimental unit.

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**LEARNING OBJECTIVE****1.13.B**

Identify experimental designs.

[Skill 2.A]

**1.13.C**

Justify the appropriateness of a particular experimental design.

[Skill 2.B]

**ESSENTIAL KNOWLEDGE****1.13.B.1**

In a completely randomized design, treatments are assigned to experimental units completely at random. Often the number of experimental units assigned to each treatment will be the same, but the sample sizes in each treatment do not have to be the same.

**1.13.B.2**

A blocking variable is a source of extraneous variation in the response variable. In a randomized block design, the experimental units are first grouped according to similar values of a blocking variable. These groups are called blocks. Units within the same block are homogeneous with respect to the blocking variable. After the blocks are formed, the treatments are randomly assigned to experimental units within each block so that all treatments occur within every block.

**1.13.B.3**

The purpose of blocking is to separate the variation in the response caused by the blocking variable from the rest of the extraneous variation in the response. Blocking allows for more precise comparisons of the response across the treatments. Within a block, the treatments can be compared without having to worry about variation in the response caused by changes in the blocking variable.

**1.13.B.4**

A matched pairs design is a randomized block design with only two treatments. Experimental units are arranged in pairs by matching on one or more extraneous sources of variation in the response variable. Each pair receives both treatments by randomly assigning one treatment to one member of the pair and the other treatment to the second member of the pair. Alternatively, each experimental unit may get both treatments while the order of the treatments is randomized.

**1.13.C.1**

One experimental design may be more appropriate than another experimental design based on the goals of the investigative study, the characteristics of the population, and the sample and variables involved.

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**LEARNING OBJECTIVE****1.13.D**

Justify the appropriateness of the conclusions based on a well-designed experiment.

**[Skill 2.B]****ESSENTIAL KNOWLEDGE****1.13.D.1**

Using random assignment of treatments to experimental units allows for cause-and-effect conclusions between the explanatory and the response variables because the potential for confounding variables is reduced.

**1.13.D.2**

Depending on the experimental unit, it may be unethical or difficult to randomly select experimental units to participate in an experiment. In that case, the study's experimental units are obtained from volunteers and will represent the population of experimental units similar to those who participated in the study.

## AP STATISTICS

### UNIT 2

# Probability, Random Variables, and Probability Distributions



**15–25%**  
AP EXAM WEIGHTING



**~24**  
CLASS PERIODS



Remember to go to [AP Classroom](#) to assign students the online **Progress Checks** for this unit.

Whether assigned as homework or completed in class, the **Progress Checks** provide each student with immediate feedback related to this unit's topics and skills.

### **Progress Check 2**

**Multiple-choice: ~42 questions**

**Free-response: 2 questions**

- Question 2: Multi-Focus on Practices 3 and 4
- Question 2: Multi-Focus on Practices 3 and 4

# Probability, Random Variables, and Probability Distributions

## ESSENTIAL QUESTIONS

- How might we represent the reading habits of people with and without an electronic reading device to help describe similarities and/or differences between the two groups?
- How can the weather be predicted?
- Why might the data we collect influence decisions being made?



## Developing Understanding

In this unit, students explore relationships in two-variable categorical data sets that will help them understand some of the rules of probability. Probabilistic reasoning allows statisticians to quantify the likelihood of random events over the long run and make statistical inferences. Simulations and concrete examples can help students understand the abstract definitions and calculations of probability. This unit builds on understandings of simulated or empirical data distributions and fundamental principles of probability to represent, interpret, and calculate parameters for theoretical probability distributions for discrete random variables. When students interpret probabilities and parameters associated with a probability distribution, they are expected to use appropriate units and relate to the context of the situation. This unit also introduces the standard normal distribution, including the empirical rule, sampling distribution of a statistic, and the central limit theorem (CLT).

## Building Statistical Practices

3.A 3.B 3.C 3.D 4.A 4.B 4.C 4.D

Probability can be a challenging topic for students because it is difficult to conceptualize future outcomes in concrete ways. Before introducing new formulas, teachers can help students get an intuitive feel for why the formulas (and related notation) make sense. For example, the probability formulas for  $P(A \text{ or } B)$  and  $P(A | B)$  can be presented intuitively with two-way tables. Simulations also help students internalize what it means to quantify random behavior. Teachers can use explicit strategies such as matching verbal scenarios to their corresponding probability formulas to help students understand when to apply different probability rules.

Students frequently misinterpret probability distributions and parameters for random variables. To prevent any misunderstanding, teachers can reinforce that a complete interpretation includes context and units. A common misconception later in the course is that every question involving probability requires a significance test. Early practice making predictions and decisions based on probability alone helps students avoid this misconception. Students can revisit these problems in later units to practice differentiating between inference and probability problems.

Statisticians use probabilities associated with the normal distribution to justify claims about populations they can never measure directly. Sketching, shading, and labeling a normal distribution aids in understanding the probability being calculated. Teachers can provide opportunities for students to practice creating graphical representations, labeling the mean, and marking off values 1, 2, and 3 standard deviations from the

mean. Students often struggle to interpret parameters of probability distributions in context—simply describing features of the graph rather than explicitly connecting those features to the situation described in the problem.

### Preparing for the AP Exam

These concepts will be evaluated using individual multiple-choice questions, a multiple-choice question set, and parts of free-response questions. To help students prepare for the AP Statistics Exam, teachers can model showing all steps in probability calculations and expect students to do the same. Calculations on the AP Statistics Exam need to include presentation of an appropriate expression that communicates the structure of the formula, substitution of relevant values extracted from the problem, and an answer. Teachers should demonstrate the expectations for distribution calculations, including identifying the distribution, labeling relevant values exacted from the problem, and indicating boundaries and direction, along with showing a correct answer.

## UNIT AT A GLANCE

Topic	Skills	Class Periods ~24 CLASS PERIODS
<b>2.1 Tabular and Graphical Representations for the Distributions of Two Categorical Variables</b>	<p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	2
<b>2.2 Summary Statistics for Two Categorical Variables</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	1
<b>2.3 Estimating Probabilities Using Simulation</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p>	1
<b>2.4 Introduction to Probability</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p>	2
<b>2.5 Mutually Exclusive Events</b>	<p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	2
<b>2.6 Conditional Probability</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p>	2
<b>2.7 Independent Events and Unions of Events</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p>	2
<b>2.8 Introduction to Random Variables and Probability Distributions</b>	<p><b>3.A</b> Construct tabular and graphical representations of data and distributions.</p>	2
<b>2.9 Parameters of Random Variables</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p>	2
<b>2.10 The Binomial Distribution</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p> <p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p>	2

(Continued)

## UNIT AT A GLANCE (cont'd)

Topic	Skills	Class Periods ~24 CLASS PERIODS
<b>2.11 The Normal Distribution</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p> <p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.C</b> Describe distributions and compare relative positions of points within a distribution.</p>	3
<b>2.12 Sampling Distributions and the Central Limit Theorem</b>	<p><b>4.C</b> Describe distributions and compare relative positions of points within a distribution.</p>	3
<p> Go to <a href="#">AP Classroom</a> to assign the <b>Progress Check</b> for Unit 2. Review the results in class to identify and address any student misunderstandings.</p>		

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit.

Activity	Topics	Sample Activity
1	2.1 2.2	<b>Quiz-Quiz-Trade</b> Give students a card containing a question with a two-way table and have them write the answer on the back. Then have them stand up and find a partner. One student quizzes the other, and then they reverse roles. Have them switch cards, find a new partner, and repeat the process.
2	2.4 2.5 2.6 2.7	<b>Think-Pair-Share</b> Provide students with a set of five probability questions: one for each of the complement rule, a mutually exclusive event, a conditional probability, the general multiplication rule, and an independent event. Ask students to individually identify the formula needed to solve each problem without doing the final calculations. Then have them share their thoughts with a partner.
3	2.10	<b>Predict and Confirm</b> Before performing a class simulation, have students discuss with their neighbor what values the distribution may take and how often they expect/predict those values to occur. For instance, what might the distribution look like for a binomial setting with $p = 0.5$ and $n = 16$ when we perform 20 trials? Have students perform the simulation and compare the results to their predictions.
4	2.11	<b>Reversing Interpretations</b> Give pairs of students four pictures of normal distributions with various parts shaded. Have students create the question that could have resulted in the picture shown. For example, if a value of 15 is labeled and the distribution is shaded to the right of 15, students could write, "What is the probability that a value is more than 15?"

## SKILLS

## 4.A

Describe and compare tabular and graphical representations of data, as well as summary statistics.

## 4.B

Justify a claim based on statistical calculations and results.

## TOPIC 2.1

# Tabular and Graphical Representations for the Distributions of Two Categorical Variables

## Required Course Content

### LEARNING OBJECTIVE

**2.1.A**

Compare tabular and graphical representations for the relationship between two categorical variables.

[Skill 4.A]

**2.1.B**

Justify a claim using tabular and graphical representations for the distributions of two categorical variables.

[Skill 4.B]

### ESSENTIAL KNOWLEDGE

**2.1.A.1**

A two-way table, also called a contingency table, can be used to summarize and compare data for two categorical variables. The entries in the cells of the table can be frequencies (i.e., counts) or relative frequencies (i.e., proportions).

**2.1.A.2**

Side-by-side bar charts, segmented bar charts, and mosaic plots are examples of graphs used to display the relationship between two categorical variables. In these graphs, the frequency or relative frequency of each category, or level, of one of the categorical variables is displayed for each category of the other categorical variable.

**2.1.A.3**

Graphical representations of two categorical variables can be used to compare the relationship of one categorical variable across the levels of the other categorical variable and determine whether the two variables are associated.

**2.1.B.1**

Tabular and graphical representations for the distributions of two categorical variables may reveal information that can be used to justify claims about the variable in context.

## TOPIC 2.2

# Summary Statistics for Two Categorical Variables

## Required Course Content

### LEARNING OBJECTIVE

**2.2.A**

Calculate summary statistics from two-way tables.

[Skill 3.B]

**2.2.B**

Compare summary statistics for two categorical variables.

[Skill 4.A]

**2.2.C**

Justify a claim using summary statistics for two categorical variables.

[Skill 4.B]

### ESSENTIAL KNOWLEDGE

**2.2.A.1**

A joint relative frequency in a two-way table is a cell frequency divided by the total for the entire table.

**2.2.A.2**

A marginal relative frequency in a two-way table is a row total divided by the total for the entire table or a column total divided by the total for the entire table.

**2.2.A.3**

A conditional relative frequency is a relative frequency computed by restricting to a particular level, or category of interest. A conditional relative frequency can be a cell frequency in a row divided by the total for that row or it can be a cell frequency in a column divided by the total for that column.

**2.2.B.1**

Summary statistics for two categorical variables can be used to compare distributions for evidence of association between the two variables.

**2.2.C.1**

Summary statistics for two categorical variables may reveal information that can be used to justify claims about the variables in context.

**SKILLS****3.B**

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

**4.A**

Describe and compare tabular and graphical representations of data, as well as summary statistics.

**4.B**

Justify a claim based on statistical calculations and results.

## SKILLS

## 3.C

Calculate and estimate expected counts, percentages, probabilities, and intervals.

## TOPIC 2.3

# Estimating Probabilities Using Simulation

## Required Course Content

### LEARNING OBJECTIVE

**2.3.A**

Estimate probabilities using simulations.

[Skill 3.C]

### ESSENTIAL KNOWLEDGE

**2.3.A.1**

A random process generates results that are determined by chance.

**2.3.A.2**

An outcome is the result of one trial of a random process.

**2.3.A.3**

An event is a collection of outcomes.

**2.3.A.4**

Simulation is a way to model random events such that simulated outcomes closely match real-world outcomes. All possible outcomes are associated with a value to be determined by chance. Record the counts of simulated outcomes and the count total.

**2.3.A.5**

The probability of an outcome or event is its long-run relative frequency—that is, its relative frequency over a large number of trials.

**2.3.A.6**

The relative frequency of an outcome or event determined from empirical data can be used to estimate the actual, or true, probability of that outcome or event.

**2.3.A.7**

The law of large numbers states that for independent trials, as the number of trials increases, the long-run relative frequency of the outcome or event gets closer and closer to a single value.

## TOPIC 2.4

# Introduction to Probability

## SKILLS

## 3.C

Calculate and estimate expected counts, percentages, probabilities, and intervals.

## Required Course Content

### LEARNING OBJECTIVE

**2.4.A**

Calculate probabilities for events and their complements.

[Skill 3.C]

### ESSENTIAL KNOWLEDGE

**2.4.A.1**

The sample space of a random process is the set of all possible nonoverlapping outcomes. The probability of the sample space is 1.

**2.4.A.2**

If all outcomes in the sample space are equally likely, then the theoretical probability an event  $E$  will occur is

$$\frac{\text{number of outcomes in event } E}{\text{total number of outcomes in the sample space}}$$

The probability of event  $E$  occurring is written as  $P(E)$ .

**2.4.A.3**

The probability of an event is a number between 0 and 1, inclusive.

**2.4.A.4**

The probability of the complement of an event  $E$ , which can be written as  $E^c$ ,  $\bar{E}$ , or  $E^c$  (i.e., the probability of “not  $E$ ”) is equal to  $1 - P(E)$ .

## SKILLS

## 4.B

Justify a claim based on statistical calculations and results.

## TOPIC 2.5

# Mutually Exclusive Events

## Required Course Content

### LEARNING OBJECTIVE

**2.5.A**

Justify why two events are mutually exclusive (or disjoint) using joint probability.

[Skill 4.B]

### ESSENTIAL KNOWLEDGE

**2.5.A.1**

The probability that events  $A$  and  $B$  both will occur, sometimes called the joint probability, is the probability of the intersection of  $A$  and  $B$ . Joint probability is defined as  $P(A \text{ intersect } B)$  or  $P(A \cap B)$ .

**2.5.A.2**

Two events are mutually exclusive, or disjoint, if they cannot occur at the same time. This means that if two events are mutually exclusive, then  $P(A \cap B) = 0$ .

## TOPIC 2.6

# Conditional Probability

## SKILLS

**3.C**

Calculate and estimate expected counts, percentages, probabilities, and intervals.

## Required Course Content

**LEARNING OBJECTIVE****2.6.A**

Calculate conditional probabilities.

[Skill 3.C]

**ESSENTIAL KNOWLEDGE****2.6.A.1**

The probability that event  $A$  will occur given that event  $B$  has occurred is called a conditional probability and is written as  $P(A | B)$ . Conditional probability is defined as

$$P(A | B) = \frac{P(A \cap B)}{P(B)}.$$

**2.6.A.2**

The general multiplication rule states that the probability that events  $A$  and  $B$  both will occur is equal to the probability that event  $A$  will occur multiplied by the conditional probability that event  $B$  will occur given that event  $A$  has occurred. The multiplication rule is defined as

$$P(A \cap B) = P(A) \cdot P(B | A).$$

## SKILLS

## 3.C

Calculate and estimate expected counts, percentages, probabilities, and intervals.

## TOPIC 2.7

# Independent Events and Unions of Events

## Required Course Content

### LEARNING OBJECTIVE

**2.7.A**

Calculate probabilities for independent events and for the union of two events.

[Skill 3.C]

### ESSENTIAL KNOWLEDGE

**2.7.A.1**

Events  $A$  and  $B$  are independent if and only if knowing whether event  $A$  has occurred (or will occur) does not change the probability that event  $B$  will occur. When events  $A$  and  $B$  are independent, then  $P(A|B) = P(A)$ ,

$$P(B|A) = P(B), \text{ and } P(A \cap B) = P(A) \cdot P(B).$$

**2.7.A.2**

The probability that event  $A$  or event  $B$  (or both) will occur is the probability of  $A$  union  $B$ . The probability of the union is defined as  $P(A \cup B)$ .

**2.7.A.3**

$$P(A \text{ union } B) = P(A) + P(B) - P(A \text{ intersect } B), \text{ or } P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

## TOPIC 2.8

# Introduction to Random Variables and Probability Distributions

## SKILLS

## 3.A

Construct tabular and graphical representations of data and distributions.

## Required Course Content

### LEARNING OBJECTIVE

**2.8.A**

Construct a probability distribution for a discrete random variable.

[Skill 3.A]

### ESSENTIAL KNOWLEDGE

**2.8.A.1**

A random variable is a variable whose values have numerical outcomes that result from a random phenomenon.

**2.8.A.2**

A probability distribution for a discrete random variable shows the probability associated with every possible value of the random variable. The sum of the probabilities over all possible values of a discrete random variable is 1.

**2.8.A.3**

A discrete probability distribution can be determined using the rules of probability or estimated with a simulation.

**2.8.A.4**

A discrete probability distribution can be represented as a graph, table, or function showing the probabilities associated with values of a random variable.

**2.8.A.5**

A cumulative probability distribution can be represented as a table or function and shows the probability of being less than or equal to each value of the discrete random variable.

## SKILLS

## 3.B

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

## 4.D

Interpret statistical calculations and results to assess meaning or a claim.

## TOPIC 2.9

# Parameters of Random Variables

## Required Course Content

### LEARNING OBJECTIVE

**2.9.A**

Calculate the mean and standard deviation for a discrete random variable.

[Skill 3.B]

### ESSENTIAL KNOWLEDGE

**2.9.A.1**

A numerical value measuring a characteristic of a probability distribution of a random variable, or a population, is a parameter. The value of a parameter is a single, fixed value.

**2.9.A.2**

The expected value (or mean) of a probability distribution is a parameter and is denoted by  $E(X)$  or  $\mu_X$ . For a discrete random variable  $X$ , the expected value is calculated as  $\mu_X = \sum x_i \cdot P(x_i)$ , where  $x_i$  is the possible value of the random variable and  $P(x_i)$  is the probability of the possible value of the random variable. The expected value can be interpreted as the long-run average outcome of the random variable. The discrete random variable can only take on values that are countable or finite.

**2.9.A.3**

The standard deviation of a probability distribution is a parameter represented by  $SD(X)$  or  $\sigma_X$ . For a discrete random variable  $X$ , the standard deviation is calculated as

$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 \cdot P(x_i)}$ , where  $x_i$  is the possible value of the random variable,  $\mu_X$  is the mean, and  $P(x_i)$  is the probability of the possible value of the random variable. The standard deviation can be interpreted as the typical deviation of the values of the random variable from the mean value (or expected value) of the random variable over the long run. The square of the standard deviation of a random variable is called the variance of the random variable and is denoted as  $V(X)$  or  $\sigma_X^2$ .

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**LEARNING OBJECTIVE**

**2.9.B**

Interpret the mean and standard deviation for a discrete random variable.

**[Skill 4.D]**

**ESSENTIAL KNOWLEDGE**

**2.9.B.1**

The mean and standard deviation for the probability distribution of a discrete random variable should be interpreted in the context of a specific population.

**SKILLS**

**3.C**

Calculate and estimate expected counts, percentages, probabilities, and intervals.

**3.D**

Calculate means, standard deviations, and parameters for probability distributions.

**4.B**

Justify a claim based on statistical calculations and results.

**4.D**

Interpret statistical calculations and results to assess meaning or a claim.

**TOPIC 2.10**

**The Binomial Distribution**

**Required Course Content**

**LEARNING OBJECTIVE**

**2.10.A**

Justify why a random variable is or is not a binomial random variable.

[Skill 4.B]

**2.10.B**

Calculate the mean and standard deviation for a binomial distribution.

[Skill 3.D]

**2.10.C**

Interpret the mean, standard deviation, and probabilities for a binomial distribution.

[Skill 4.D]

**2.10.D**

Estimate probabilities of binomial random variables using data from a simulation.

[Skill 3.C]

**ESSENTIAL KNOWLEDGE**

**2.10.A.1**

A binomial random variable,  $X$ , is a discrete random variable that counts the number of successes in repeated independent trials,  $n$ , that have only two possible outcomes (success or failure), with the probability of success  $p$  and the probability of failure  $1 - p$ .

**2.10.B.1**

If a random variable is binomial, its mean,  $\mu_X$ , is  $np$  and its standard deviation,  $\sigma_X$ , is  $\sqrt{np(1-p)}$ .

**2.10.C.1**

The mean, standard deviation, and probabilities for a binomial distribution should be interpreted in context.

**2.10.D.1**

A probability distribution can be constructed using the rules of probability or estimated with a simulation.

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**LEARNING OBJECTIVE****2.10.E**

Calculate probabilities for a binomial distribution.

**[Skill 3.C]****ESSENTIAL KNOWLEDGE****2.10.E.1**

The probability that a binomial random variable,  $X$ , has exactly  $x$  successes for  $n$  independent trials, when the probability of success is  $p$ , is

calculated as  $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$ ,

$x = 0, 1, 2, \dots, n$ . This is called the binomial probability function.

## SKILLS

## 3.C

Calculate and estimate expected counts, percentages, probabilities, and intervals.

## 3.D

Calculate means, standard deviations, and parameters for probability distributions.

## 4.C

Describe distributions and compare relative positions of points within a distribution.

## TOPIC 2.11

# The Normal Distribution

## Required Course Content

### LEARNING OBJECTIVE

**2.11.A**

Describe a normal distribution.

[Skill 4.C]

**2.11.B**

Calculate the mean and standard deviation for a normal distribution.

[Skill 3.D]

### ESSENTIAL KNOWLEDGE

**2.11.A.1**

A continuous random variable is a variable that can take on any value within a specified domain. Every interval within the domain has a probability associated with it.

**2.11.A.2**

Many continuous random variables are well-modeled by a normal distribution.

**2.11.A.3**

A normal distribution can be described as a continuous, unimodal, bell-shaped, and symmetric curve.

**2.11.A.4**

A normal curve can be used to model a distribution of data and a continuous random variable.

**2.11.A.5**

The normal distribution, or the normal curve, is identified by two parameters, the mean,  $\mu$ , and the standard deviation,  $\sigma$ . The smaller the standard deviation, the taller and more concentrated the normal curve is around its mean. The larger the standard deviation, the shorter and less concentrated the normal curve is around its mean.

**2.11.B.1**

A standard normal distribution is a normal distribution with mean  $\mu = 0$  and standard deviation  $\sigma = 1$ .

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**LEARNING OBJECTIVE****2.11.C**

Calculate percentages from a normal distribution using the empirical rule.

**[Skill 3.C]**

**2.11.D**

Calculate the probability that a particular value lies within a given interval of a normal distribution.

**[Skill 3.C]**

**2.11.E**

Calculate the associated intervals and areas of a normal distribution.

**[Skill 3.C]**

**ESSENTIAL KNOWLEDGE****2.11.C.1**

The empirical rule can be used to estimate the area of a region under the graph of the normal distribution curve. For a normal distribution, approximately 68% of observations are within 1 standard deviation of the mean, approximately 95% of observations are within 2 standard deviations of the mean, and approximately 99.7% of observations are within 3 standard deviations of the mean. This is called the empirical rule, or the 68–95–99.7 rule.

**2.11.D.1**

If the distribution of a random variable is approximately normal, the probability that the random variable takes on values within a particular interval of the random variable is determined by the area under the normal curve within that interval. The total probability or area under the normal curve is 1.

**2.11.E.1**

The boundaries of an interval associated with a given area in a normal distribution can be determined using technology or using z-scores and a standard normal table.

**2.11.E.2**

Intervals associated with a given area in a normal distribution can be determined by assigning appropriate inequalities to the boundaries of the intervals. To determine the intervals,  $p$  is defined as a number between 0 and 100,  $x_a$  is the lower bound, and  $x_b$  is the upper bound on a normal distribution.

**2.11.E.2.i**

$P(X < x_a) = \frac{p}{100}$  means that the lowest  $p\%$  of the values lie to the left of  $x_a$ .

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**LEARNING OBJECTIVE**

**2.11.E**

Calculate the associated intervals and areas of a normal distribution.

[Skill 3.C]

**2.11.F**

Compare measures of relative position for distributions.

[Skill 4.C]

**ESSENTIAL KNOWLEDGE**

**2.11.E.2.ii**

$P(x_a < X < x_b) = \frac{p}{100}$  means that  $p\%$  of the values lie between  $x_a$  and  $x_b$ .

**2.11.E.2.iii**

$P(X > x_b) = \frac{p}{100}$  means that the highest  $p\%$  of the values lie to the right of  $x_b$ .

**2.11.E.2.iv**

To determine the most extreme  $p\%$  of values on both sides requires dividing the area associated with  $p\%$  into two equal areas on either extreme of the distribution:

$$P(X < x_a) = \frac{1}{2} \left( \frac{p}{100} \right) \text{ and } P(X > x_b) = \frac{1}{2} \left( \frac{p}{100} \right)$$

mean that half of the  $p\%$  most extreme values lie to the left of  $x_a$  and half of the  $p\%$  most extreme values lie to the right of  $x_b$ .

**2.11.F.1**

Percentiles and proportions may be used to compare relative positions of individual values within a normal distribution or between normal distributions.

## TOPIC 2.12

# Sampling Distributions and the Central Limit Theorem

## SKILLS

## 4.C

Describe distributions and compare relative positions of points within a distribution.

## Required Course Content

### LEARNING OBJECTIVE

**2.12.A**

Describe sampling distributions with simulations.

[Skill 4.C]

### ESSENTIAL KNOWLEDGE

**2.12.A.1**

A sampling distribution of a statistic is the distribution of values of the statistic for all possible samples of a given size from a given population.

**2.12.A.2**

The sampling distribution of a statistic can be simulated by repeatedly generating a large number of random samples from the population assuming known value(s) for the parameter(s). The value of the statistic is determined and recorded for each sample. The resulting distribution of the sample statistic values approximates the sampling distribution of the statistic.

**2.12.A.3**

A randomization distribution is the distribution of a statistic generated by simulation from repeatedly randomly reallocating, or reassigning, the response values to treatment groups. The value of the statistic is determined and recorded for each reallocation, or reassignment. The resulting distribution of the statistic values approximates the sampling distribution of the statistic.

**2.12.A.4**

The central limit theorem (CLT) states that the sampling distribution of a mean of a random sample has a shape that can be approximated by a normal distribution. The larger the sample is, the better the approximation will be.

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**AP STATISTICS**

**UNIT 3**

# **Inference for Categorical Data: Proportions**



**15–25%**  
AP EXAM WEIGHTING



**~30**  
CLASS PERIODS



Remember to go to [AP Classroom](#) to assign students the online **Progress Checks** for this unit.

Whether assigned as homework or completed in class, the **Progress Checks** provide each student with immediate feedback related to this unit's topics and skills.

### **Progress Check 3**

**Multiple-choice: ~65 questions**

**Free-response: 3 questions**

- Question 3: Inference
- Question 3: Inference
- Question 3: Inference

# Inference for Categorical Data: Proportions

## ESSENTIAL QUESTIONS

- How can we measure changing opinions in populations over time?
- When can we perform inference calculations using data from websites to answer everyday questions?
- How can we determine whether the difference between two groups is statistically significant?
- Are the distributions of various types of wild birds different in urban areas than in rural areas?



## Developing Understanding

This unit introduces sampling distributions and statistical inference. Students learn that sample statistics can be used to estimate corresponding population parameters and that measures of center (mean) and variability (standard deviation) for these sampling distributions can be determined directly from the population parameters when certain sampling criteria are met. For large enough samples from any population, these sampling distributions can be approximated by a normal distribution. Simulating sampling distributions helps students understand how the values of statistics vary in repeated random sampling from populations with known parameters.

This unit also introduces statistical inference. Students analyze categorical data to make inferences about binomial population proportions. Provided that conditions are met, students use statistical inference to construct and interpret confidence intervals to estimate population proportions and perform significance tests to evaluate claims about population proportions. Students begin by learning inference procedures for one proportion and then examine inference methods for a difference between two proportions. They also interpret the two types of errors that can be made in a significance test, the probability of each error occurring, and possible consequences, in context.

Students are also introduced to chi-square tests, which can be used when there are two or more categories. Students need to understand how to select between the following two tests: (1) the chi-square test for independence (for associations between categorical variables within a single population), and (2) the chi-square test for homogeneity (for comparing distributions of a categorical variable across populations or treatments). To integrate conceptual understanding, teachers can help students make connections among frequency tables, conditional probability, and calculating expected counts. The chi-square statistic is introduced to measure the distance between observed and expected counts relative to expected counts.

## Building Statistical Practices

2.C 2.D 2.E 3.C 3.D 3.E 4.B 4.C 4.D 4.E 4.F 4.G

Unit 3 is a critical transition point in the course, as students begin learning skills that will be applied repeatedly in subsequent units. Students build proficiency over time by familiarizing themselves with procedures. Applying different inference methods requires fluency with verifying conditions. Students often do not earn points because they check conditions superficially without explicitly connecting them to the problem. For example, students may write “SRS” when they should instead explain that a simple random sample was taken. Teachers can offer opportunities for students to practice verifying conditions in context by providing numerical calculations and explaining how each condition is met. Precision of language is key. Students often interpret confidence intervals and confidence levels incorrectly. Providing students with sentence starters or templates helps them learn to generate appropriate responses. For example, a sentence starter for a confidence interval might be, “We are 95% confident that the interval from  $a$  to  $b$  captures the [parameter in context].”

For decisions based on a hypothesis test, students may incorrectly claim that “we can accept” or “we have proven” the null. Teachers can reinforce early and often that statistical tests do not provide evidence for what can be accepted or proven; they only provide evidence for rejecting or failing to reject the null.

Students continue to apply the same problem-solving structure to chi-square hypothesis testing by stating the hypotheses in words, explicitly identifying the correct procedure, verifying conditions, calculating the test statistic and the  $p$ -value, and then drawing a conclusion in context that is directly linked to the  $p$ -value. Students will benefit from opportunities to practice the distinctive elements for each type of chi-square test, including analysis of expected counts, degrees of freedom, verbally stated hypotheses, and two-way tables. When the  $p$ -value is large, drawing an appropriate conclusion can be challenging for students. Saying there is no association between two variables is equivalent to incorrectly accepting the null hypothesis. Instead, teachers can show students how to use nondefinitive language in their conclusions, such as, “The data do not provide strong enough evidence to conclude that the variables are associated.” Frequent opportunities to practice writing and give detailed feedback will help students improve.

### Preparing for the AP Exam

These concepts will be evaluated using individual multiple-choice questions and parts of free-response questions. Responses on the AP Statistics Exam often uncover gaps in students’ understanding of sampling distributions. Students are expected to clearly communicate whether they are talking about the distribution of a population, a sample of values (e.g., heights), or a sample statistic from repeated samples (e.g., mean heights). Broad generalizations, such as “larger samples have less variability,” leave the exam reader unsure whether the student is referring to variability within a sample (for which the statement would be false) or a sampling distribution. The word *it* often introduces ambiguity to a response. Students frequently show confusion about what condition to check when asserting that the sampling distribution of a given statistic is approximately normal.

When using statistical inference to construct confidence intervals or perform hypothesis tests for proportions, students are expected to identify the appropriate inference method by name or formula. For inference with population proportions, students need to verify that the following three conditions are met: (1) the randomization condition, (2) the 10% condition (when sampling without replacement), and (3) the normality condition. Simple and specific verification is best. Students then present calculations and interpret results in the context of the problem. Students often find it beneficial to use the language provided in the question.

When writing hypotheses for chi-square tests, students are expected to refer to the population by using language from the question, for example, “The null hypothesis is that the grade level and club participation are independent (i.e., they are not associated) for the population of students at the local high school.” As always, a complete response includes the name of the test and provides evidence verifying appropriate conditions. For chi-square tests, the conditions are (1) the randomization condition, (2) the 10% condition (when sampling without replacement), and (3) the expected values condition. Precise student answers state that all expected counts (rather than actual counts) are at least 5. Students need to clearly present calculations and state the conclusion in context, with linkage to  $p$ -values, and avoid tacitly accepting the null hypothesis. If the  $p$ -value is greater than conventional significance levels, the correct conclusion of a chi-square test for independence is that there is insufficient evidence of an association.

## UNIT AT A GLANCE

Topic	Skills	Class Periods
<b>3.1 Estimators</b>	<p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	2
<b>3.2 Sampling Distributions for Sample Proportions</b>	<p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	1
<b>3.3 Constructing a Confidence Interval for a Population Proportion</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>3.4 Justifying a Claim Based on a Confidence Interval for a Population Proportion</b>	<p><b>2.D</b> Identify types of errors and relationships among components in statistical inference methods.</p> <p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>3.5 Setting Up a Test for a Population Proportion</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>2.E</b> Identify the null and alternative hypotheses.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>3.6 <math>p</math>-Values</b>	<p><b>4.F</b> Interpret results of statistical inference methods.</p>	2
<b>3.7 Carrying Out a Test for a Population Proportion</b>	<p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>3.8 Potential Errors When Performing Tests</b>	<p><b>2.D</b> Identify types of errors and relationships among components in statistical inference methods.</p> <p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p>	2

(Continued)

## UNIT AT A GLANCE (cont'd)

Topic	Skills	Class Periods
<b>3.9 Sampling Distributions for the Difference Between Sample Proportions</b>	<p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	1
<b>3.10 Constructing a Confidence Interval for the Difference Between Two Population Proportions</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>3.11 Justifying a Claim Based on a Confidence Interval for the Difference Between Two Population Proportions</b>	<p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>3.12 Setting Up a Test for the Difference Between Two Population Proportions</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>2.E</b> Identify the null and alternative hypotheses.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>3.13 Carrying Out a Test for the Difference Between Two Population Proportions</b>	<p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>3.14 Setting Up a Chi-Square Test for Homogeneity or Independence</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>2.E</b> Identify the null and alternative hypotheses.</p> <p><b>4.C</b> Describe distributions and compare relative positions of points within a distribution.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	3
<b>3.15 Carrying Out a Chi-Square Test for Homogeneity or Independence</b>	<p><b>3.C</b> Calculate and estimate expected counts, percentages, probabilities, and intervals.</p> <p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	3



Go to [AP Classroom](#) to assign the **Progress Check** for Unit 3. Review the results in class to identify and address any student misunderstandings.

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit.

Activity	Topics	Sample Activity
1	3.5 3.12 3.14	<p><b>Error Analysis</b></p> <p>Give student pairs a worksheet with 20 sets of hypotheses, each with a common student mistake. Be sure to include hypotheses for a test for a population proportion, a test for the difference between two proportions, a chi-square test for independence, and a chi-square test for homogeneity. Have students circle the incorrect part, write why the circled component is incorrect, and then write the correct hypotheses. Include errors such as using statistics instead of parameters and interchanging the = and &gt; in the two hypotheses.</p>
2	3.7 3.13 3.15	<p><b>Sentence Starters</b></p> <p>For a given question, provide students with a set of hypotheses, <math>p</math>-value, significance level, and context. Have them compare the <math>p</math>-value to the significance level to determine whether to reject the null hypothesis. Use a sentence starter with blanks to fill in, and have students write a sentence in context explaining whether they have enough evidence to reject <math>H_0</math> or if they will fail to reject <math>H_0</math>. Make sure students avoid the common mistake of implying that evidence supports an accept <math>H_0</math> conclusion or a reject <math>H_0</math> conclusion.</p>
3	3.4 3.11	<p><b>The Scribe and the Calculator</b></p> <p>Have students work with a partner to construct and interpret a confidence interval for a population proportion. Only one partner is allowed to use the calculator, and only the other partner is allowed to write. When a calculation needs to be made, only the scribe can describe to the calculator operator what buttons to push; when writing needs to be done, only the calculator operator can describe to the scribe what needs to be written. Have students switch roles when constructing and interpreting a confidence interval for the difference of two population proportions.</p>

## SKILLS

## 3.D

Calculate means, standard deviations, and parameters for probability distributions.

## 4.B

Justify a claim based on statistical calculations and results.

## TOPIC 3.1

# Estimators

### Required Course Content

#### LEARNING OBJECTIVE

**3.1.A**

Justify why an estimator is or is not unbiased.

[Skill 4.B]

**3.1.B**

Calculate estimates for a population parameter.

[Skill 3.D]

#### ESSENTIAL KNOWLEDGE

**3.1.A.1**

When estimating a population parameter, an estimator is unbiased if, on average, the value of the estimator does not underestimate or overestimate the population parameter.

**3.1.B.1**

A sample statistic is a point estimator of the corresponding population parameter and can be thought of as the estimate of the population parameter. For example, the sample proportion  $\hat{p}$  is a point estimator for the population proportion  $p$ .

TOPIC 3.2

# Sampling Distributions for Sample Proportions

## Required Course Content

### LEARNING OBJECTIVE

**3.2.A**

Calculate the mean and standard deviation of a sampling distribution for a sample proportion.

[Skill 3.D]

**3.2.B**

Justify the appropriateness of conditions for the sampling distribution of a sample proportion.

[Skill 4.E]

**3.2.C**

Interpret the mean, standard deviation, and probabilities for a sampling distribution of a sample proportion.

[Skill 4.D]

### ESSENTIAL KNOWLEDGE

**3.2.A.1**

For a population with population proportion  $p$ , when the sampled values are independent, the sampling distribution of a sample proportion  $\hat{p}$  has a mean  $\mu_{\hat{p}} = p$  and a standard deviation

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

**3.2.B.1**

Sampling without replacement requires that two conditions be met:

**3.2.B.1.i**

The randomization condition—the data should be collected using a random sample.

**3.2.B.1.ii**

The 10% condition—the population size must be at least 10 times larger than the sample size ( $n \leq 10\%N$ ), where  $N$  is the size of the population and  $n$  is the sample size.

**3.2.B.2**

The sampling distribution of the sample proportion  $\hat{p}$  is approximately normal provided the sample size is large enough. To ensure the sample size is large enough, the following condition must be met:  $np \geq 10$  and  $n(1-p) \geq 10$ , where  $np$  is the expected number of successes and  $n(1-p)$  is the expected number of failures.

**3.2.C.1**

The mean, standard deviation, and probabilities for a sampling distribution of a sample proportion should be interpreted in the context of a specific population.

SKILLS

**3.D**

Calculate means, standard deviations, and parameters for probability distributions.

**4.D**

Interpret statistical calculations and results to assess meaning or a claim.

**4.E**

Justify the use of a chosen statistical inference method by verifying conditions.

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 3.E

Calculate appropriate statistical inference method results.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 3.3

# Constructing a Confidence Interval for a Population Proportion

## Required Course Content

### LEARNING OBJECTIVE

**3.3.A**

Identify an appropriate confidence interval procedure including the parameter for a population proportion.

[Skill 2.C]

**3.3.B**

Justify the appropriateness of constructing a confidence interval for a population proportion by verifying conditions.

[Skill 4.E]

### ESSENTIAL KNOWLEDGE

**3.3.A.1**

A confidence interval is an interval estimate for a population parameter. Based on the sample proportion, a confidence interval can be calculated to estimate the value of a single population proportion. The appropriate confidence interval procedure is a one-sample z-interval for a population proportion.

**3.3.A.2**

The parameter for a confidence interval for a population proportion should reference the proportion, the response variable, and the population in context.

**3.3.B.1**

A one-sample z-interval for a population proportion requires that three conditions be met:

**3.3.B.1.i**

The randomization condition—the data should be collected using a random sample.

**3.3.B.1.ii**

The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size ( $n \leq 10\%N$ ), where  $N$  is the size of the population and  $n$  is the sample size.

**3.3.B.1.iii**

The normality condition—the observed number of successes,  $n\hat{p}$ , and the observed number of failures,  $n(1-\hat{p})$ , should be at least 10.

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**LEARNING OBJECTIVE**

**3.3.C**

Calculate an appropriate confidence interval for a population proportion.

[Skill 3.E]

**3.3.D**

Calculate the standard error and margin of error of a sample statistic for a confidence interval for a population proportion, and estimate a given sample size from the margin of error.

[Skill 3.E]

**ESSENTIAL KNOWLEDGE**

**3.3.C.1**

$z^*$  denotes a critical value, such that  $-z^*$  and  $+z^*$  represent the boundaries enclosing the middle  $C\%$  of the standard normal distribution, in which  $C\%$  is an approximate confidence level with which the population proportion is estimated.

**3.3.C.2**

An interval estimate can be constructed as point estimate  $\pm$  (margin of error). For a population proportion, the one-sample

$z$ -interval estimate is  $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ .

**3.3.D.1**

The standard error ( $SE$ ) of a statistic is an estimate of the standard deviation of the sampling distribution of the statistic. The standard error of the sample proportion  $\hat{p}$  is

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

**3.3.D.2**

The standard error quantifies the typical amount that a statistic will vary from the value of the corresponding population parameter.

**3.3.D.3**

The margin of error of  $\hat{p}$  is half the width of the confidence interval and is calculated as the critical value ( $z^*$ ) times the standard error ( $SE$ )

of  $\hat{p}$ , which equals  $z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ .

**3.3.D.4**

The formula for the margin of error ( $MOE$ ) can be rearranged to solve for  $n$ ,

$$n = \frac{(z^*)^2 (\hat{p})(1-\hat{p})}{(MOE)^2}, \text{ the minimum sample}$$

needed to achieve a given margin of error. For this purpose, if  $\hat{p}$  is not defined or unable to be calculated, use  $\hat{p} = 0.5$  in order to find the upper bound for the sample size that will result in a given margin of error.

## SKILLS

## 2.D

Identify types of errors and relationships among components in statistical inference methods.

## 4.F

Interpret results of statistical inference methods.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 3.4

# Justifying a Claim Based on a Confidence Interval for a Population Proportion

## Required Course Content

### LEARNING OBJECTIVE

**3.4.A**

Interpret a confidence interval in context for a population proportion.

[Skill 4.F]

**3.4.B**

Justify a claim based on a confidence interval for a population proportion.

[Skill 4.G]

### ESSENTIAL KNOWLEDGE

**3.4.A.1**

Because the confidence interval for a population proportion is calculated based on a sample from a population, the computed interval may or may not contain the value of the population proportion.

**3.4.A.2**

The interpretation of the confidence level is that in repeated random sampling with the same sample size, approximately  $C\%$  of confidence intervals calculated will capture the population proportion, with  $C$  representing the numerical value of the confidence level used.

**3.4.A.3**

When interpreting a  $C\%$  confidence interval for a population proportion, we say we are  $C\%$  confident that the interval  $(a, b)$  contains the true value of the parameter for the population, where  $a$  represents the lower limit and  $b$  represents the upper limit. An interpretation of a confidence interval for a population proportion includes a reference to the parameter with details about the population it represents in the context of the study.

**3.4.B.1**

A confidence interval for a population proportion provides a range of plausible values that may serve as convincing evidence to support a particular claim about the population proportion.

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**LEARNING OBJECTIVE****3.4.C**

Identify the relationships among sample size, confidence interval width, confidence level, and margin of error for a population proportion.

**[Skill 2.D]**

**ESSENTIAL KNOWLEDGE****3.4.C.1**

For a given sample, increasing the confidence level will result in the following:

**3.4.C.1.i**

The critical value will increase.

**3.4.C.1.ii**

The margin of error will increase.

**3.4.C.1.iii**

The width of the confidence interval will increase.

**3.4.C.2**

Increasing the sample size decreases the standard error. Thus, when all other things remain the same, the width of the confidence interval for a population proportion tends to decrease as the sample size increases. For a confidence interval for a population proportion with a given confidence level, the width of the interval is approximately proportional to  $\frac{1}{\sqrt{n}}$ .

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 2.E

Identify the null and alternative hypotheses.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 3.5

# Setting Up a Test for a Population Proportion

## Required Course Content

### LEARNING OBJECTIVE

**3.5.A**

Identify an appropriate testing method for a population proportion including the parameter for the population proportion.

[Skill 2.C]

**3.5.B**

Identify the null and alternative hypotheses for a population proportion.

[Skill 2.E]

### ESSENTIAL KNOWLEDGE

**3.5.A.1**

A hypothesis test is a statistical inference procedure that is used to make a decision about the value of a population parameter. The appropriate hypothesis testing procedure is a one-sample z-test for a population proportion.

**3.5.A.2**

The parameter for a hypothesis test for a population proportion should reference the population parameter, the response variable, and the population in context.

**3.5.B.1**

In the hypothesis testing procedure, the null hypothesis,  $H_0$ , is the statement about a parameter that is assumed to be correct unless there is convincing statistical evidence suggesting otherwise. It is the status quo condition. The alternative hypothesis,  $H_a$ , is the claim or belief about a parameter for which evidence is being collected. A researcher's claim or belief about the population parameter is represented by the alternative hypothesis.

**3.5.B.2**

The null hypothesis contains an equality reference ( $=$ ,  $\geq$ , or  $\leq$ ). Although the null hypothesis for a one-sided test may include an inequality symbol, in AP Statistics it is tested at the boundary of equality. The alternative hypothesis with  $<$  or  $>$  is called one-sided, and the alternative hypothesis with  $\neq$  is called two-sided.

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**LEARNING OBJECTIVE**

**3.5.B**

Identify the null and alternative hypotheses for a population proportion.

[Skill 2.E]

**3.5.C**

Justify the appropriateness of a hypothesis test for a population proportion by verifying conditions.

[Skill 4.E]

**ESSENTIAL KNOWLEDGE**

**3.5.B.3**

The null hypothesis for a one-sample z-test for a population proportion is as follows:  $H_0: p = p_0$ , where  $p_0$  is the null hypothesized value for the population proportion. A one-sided alternative hypothesis for a one-sample z-test for a population proportion is either  $H_a: p < p_0$  or  $H_a: p > p_0$ . A two-sided alternative hypothesis is  $H_a: p \neq p_0$ .

**3.5.C.1**

A one-sample z-test for a population proportion requires that three conditions be met:

**3.5.C.1.i**

The randomization condition—the data should be collected using a random sample.

**3.5.C.1.ii**

The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size ( $n \leq 10\%N$ ), where  $N$  is the size of the population and  $n$  is the sample size.

**3.5.C.1.iii**

The normality condition—the expected number of successes,  $np_0$ , and the expected number of failures,  $n(1 - p_0)$ , should be at least 10.

## SKILLS

## 4.F

Interpret results of statistical inference methods.

## TOPIC 3.6

# $p$ -Values

### Required Course Content

#### LEARNING OBJECTIVE

**3.6.A**

Interpret the  $p$ -value of a hypothesis test for a population proportion.

[Skill 4.F]

#### ESSENTIAL KNOWLEDGE

**3.6.A.1**

Given the null hypothesis is true, there is a probability distribution of the test statistic called the null distribution. Using the null distribution, the  $p$ -value is the probability of obtaining a test statistic as extreme or more extreme (i.e., in the direction of the alternative hypothesis) than the test statistic that is observed given that the null hypothesis is true. That is, when  $x$  is the test statistic, the  $p$ -value is determined by finding the following:

**3.6.A.1.i**

The probability at or above the observed value of the test statistic ( $P(z \geq x)$ ), if the alternative is  $>$

**3.6.A.1.ii**

The probability at or below the observed value of the test statistic ( $P(z \leq x)$ ), if the alternative is  $<$

**3.6.A.1.iii**

The probability less than or equal to the negative of the absolute value of the test statistic plus the probability greater than or equal to the absolute value of the test statistic, ( $P(z \leq -|x|) + P(z \geq |x|)$ ), if the alternative is  $\neq$

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## LEARNING OBJECTIVE

**3.6.A**

Interpret the  $p$ -value of a hypothesis test for a population proportion.

[Skill 4.F]

## ESSENTIAL KNOWLEDGE

**3.6.A.2**

If the distribution of the test statistic has been simulated, the  $p$ -value is the proportion of values in the null distribution that are as extreme or more extreme than the observed value of the test statistic. This is as follows:

**3.6.A.2.i**

The proportion at or above the observed value of the test statistic, if the alternative is  $>$

**3.6.A.2.ii**

The proportion at or below the observed value of the test statistic, if the alternative is  $<$

**3.6.A.2.iii**

The proportion less than or equal to the negative of the absolute value of the test statistic plus the proportion greater than or equal to the absolute value of the test statistic, if the alternative is  $\neq$

**3.6.A.3**

An interpretation of the  $p$ -value of a hypothesis test for a population proportion should include a statement that the  $p$ -value is computed by assuming the null hypothesis is true (i.e., by assuming the true population proportion is equal to the particular value stated in the null hypothesis in context).

**3.6.A.4**

Small  $p$ -values indicate that the observed value of the test statistic would be unusual if the null hypothesis were true and therefore provide evidence for the alternative hypothesis. The lower the  $p$ -value, the more convincing the statistical evidence for the alternative hypothesis.

**3.6.A.5**

$p$ -values that are not small indicate that the observed value of the test statistic would not be unusual if the null hypothesis were true and therefore do not provide convincing statistical evidence for the alternative hypothesis, nor do they provide evidence that the null hypothesis is true.

## SKILLS

## 3.E

Calculate appropriate statistical inference method results.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 3.7

# Carrying Out a Test for a Population Proportion

## Required Course Content

### LEARNING OBJECTIVE

**3.7.A**

Calculate an appropriate test statistic and  $p$ -value for testing a hypothesis about a population proportion.

[Skill 3.E]

**3.7.B**

Justify a claim about the population based on the results of a hypothesis test for a population proportion.

[Skill 4.G]

### ESSENTIAL KNOWLEDGE

**3.7.A.1**

The test statistic for testing a population

$$\text{proportion is } z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

The  $z$ -statistic has a standard normal distribution when the null hypothesis is true.

**3.7.A.2**

The distribution of the test statistic assuming the null hypothesis is true (null distribution) can be approximated by a probability model (e.g., a theoretical distribution such as the standard normal distribution).

**3.7.A.3**

The  $p$ -value of a one-sample  $z$ -test for a population proportion is found from the standard normal distribution using a table or technology.

**3.7.B.1**

The significance level of a hypothesis test, denoted by  $\alpha$ , is the predetermined probability of rejecting the null hypothesis given that it is true. The significance level may be given or determined by the researcher. The relationship between a  $p$ -value and the significance level of a hypothesis test determines whether a result is statistically significant.

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**LEARNING OBJECTIVE****3.7.B**

Justify a claim about the population based on the results of a hypothesis test for a population proportion.

[Skill 4.G]

**ESSENTIAL KNOWLEDGE****3.7.B.2**

A formal decision in a hypothesis test explicitly compares the  $p$ -value to the significance level,  $\alpha$ . If the  $p$ -value  $\leq \alpha$ , then reject the null hypothesis,  $H_0: p = p_0$ . If the  $p$ -value  $> \alpha$ , then fail to reject the null hypothesis.

**3.7.B.3**

Rejecting the null hypothesis means there is convincing statistical evidence to support the alternative hypothesis. Failing to reject the null hypothesis means there is not convincing statistical evidence to support the alternative hypothesis.

**3.7.B.4**

A hypothesis test can lead to rejecting or not rejecting the null hypothesis but can never lead to concluding or proving that the null hypothesis is true. Lack of statistical evidence for the alternative hypothesis is not the same as evidence for the null hypothesis.

**3.7.B.5**

The results of a hypothesis test for a population proportion can serve as the statistical reasoning to support the answer to an investigative question about the population that was sampled.

**3.7.B.6**

A conclusion for the hypothesis test for a population proportion is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the parameter and the population.

**SKILLS**

**2.D**

Identify types of errors and relationships among components in statistical inference methods.

**3.C**

Calculate and estimate expected counts, percentages, probabilities, and intervals.

**4.D**

Interpret statistical calculations and results to assess meaning or a claim.

**TOPIC 3.8**

# Potential Errors When Performing Tests

## Required Course Content

**LEARNING OBJECTIVE**

**3.8.A**

Identify Type I and Type II errors.

[Skill 2.D]

**3.8.B**

Calculate the probability of Type I and Type II errors.

[Skill 3.C]

**ESSENTIAL KNOWLEDGE**

**3.8.A.1**

A Type I error occurs when there is convincing statistical evidence that the alternative hypothesis is true (due to the small  $p$ -value), but it is not.

**3.8.A.2**

A Type II error occurs when there is not convincing statistical evidence that the alternative hypothesis is true (due to the large  $p$ -value), but it is.

**3.8.A.3**

The power of a hypothesis test is the probability that a hypothesis test will correctly reject the false null hypothesis.

**3.8.B.1**

The probability of making a Type I error is defined as the significance level,  $\alpha$ . For a given study and hypothesis test, the probability of making a Type I error is typically set to a small value (e.g., 0.01, 0.05, 0.10) prior to collecting the data.

**3.8.B.2**

The probability of making a Type II error is  $1 - \text{power}$ .

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**LEARNING OBJECTIVE**

**3.8.C**

Identify the factors that affect the probability of errors in hypothesis testing.

**[Skill 2.D]**

**3.8.D**

Interpret Type I and Type II errors.

**[Skill 4.D]**

**ESSENTIAL KNOWLEDGE**

**3.8.C.1**

For a given study and hypothesis test, the probability of a Type II error should ideally be small, and thus, the power will be large (e.g.,  $P(\text{Type II error}) = 0.20$  and power = 0.80). The probability of a Type II error decreases and the power increases when any one of the following occurs, provided the others do not change:

**3.8.C.1.i**

Sample size(s) increases.

**3.8.C.1.ii**

Standard error decreases.

**3.8.C.1.iii**

True parameter value is farther from the null hypothesis.

**3.8.C.1.iv**

Significance level ( $\alpha$ ) of a test increases.

**3.8.D.1**

In some studies, making a Type I error may have more serious consequences than making a Type II error. In other studies, making a Type II error may have more serious consequences than making a Type I error. The consequences of each error should be considered prior to conducting the study.

**3.8.D.2**

Because the significance level,  $\alpha$ , is the probability of making a Type I error, the consequences of a Type I error influence decisions about a significance level.

**3.8.D.3**

Because sample size influences the probability of making a Type II error, the consequences of a Type II error influence decisions about how large the sample size should be.

## SKILLS

## 3.D

Calculate means, standard deviations, and parameters for probability distributions.

## 4.D

Interpret statistical calculations and results to assess meaning or a claim.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 3.9

# Sampling Distributions for the Difference Between Sample Proportions

## Required Course Content

### LEARNING OBJECTIVE

## 3.9.A

Calculate the mean and standard deviation of the sampling distribution for the difference between two sample proportions.

[Skill 3.D]

## 3.9.B

Justify the appropriateness of conditions for the sampling distribution for the difference between two sample proportions.

[Skill 4.E]

### ESSENTIAL KNOWLEDGE

## 3.9.A.1

For two independent populations, with population proportions  $p_1$  and  $p_2$ , when the sampled values are independent, the sampling distribution for the difference in sample proportions,  $\hat{p}_1 - \hat{p}_2$ , has a mean,  $\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$ , and standard deviation,

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

## 3.9.B.1

When sampling without replacement, two conditions must be met:

## 3.9.B.1.i

The randomization condition—the data should be collected using two independent random samples.

## 3.9.B.1.ii

The 10% condition—the size of each sample should be less than or equal to 10% of the respective population size:  $n_1 \leq 10\%N_1$  and  $n_2 \leq 10\%N_2$ , where  $N_1$  is the size of population 1 and  $N_2$  is the size of population 2. The sample sizes are represented as  $n_1$  and  $n_2$ .

## 3.9.B.2

If the data come from an experiment, the data only need to meet the randomization condition. The treatments must be randomly assigned to the experimental units to meet the randomization condition.

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**LEARNING OBJECTIVE**

**3.9.B**

Justify the appropriateness of conditions for the sampling distribution for the difference between two sample proportions.

[Skill 4.E]

**3.9.C**

Interpret the mean, standard deviation, and probabilities for the sampling distribution for the difference between two sample proportions.

[Skill 4.D]

**ESSENTIAL KNOWLEDGE**

**3.9.B.3**

The sampling distribution for the difference between sample proportions,  $\hat{p}_1 - \hat{p}_2$ , will have an approximately normal distribution provided both sample sizes are large enough. To ensure that both samples are large enough, the data must meet the following conditions:  $n_1 p_1 \geq 10$ ,  $n_1(1 - p_1) \geq 10$ ,  $n_2 p_2 \geq 10$ , and  $n_2(1 - p_2) \geq 10$ , where  $n_1 p_1$  and  $n_2 p_2$  are the expected number of successes and  $n_1(1 - p_1)$  and  $n_2(1 - p_2)$  are the expected number of failures.

**3.9.C.1**

The mean, standard deviation, and probabilities for the sampling distribution for the difference between two sample proportions should be interpreted within the context of two specific populations.

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 3.E

Calculate appropriate statistical inference method results.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 3.10

# Constructing a Confidence Interval for the Difference Between Two Population Proportions

## Required Course Content

### LEARNING OBJECTIVE

**3.10.A**

Identify an appropriate confidence interval procedure including the parameters for the difference between two population proportions.

[Skill 2.C]

**3.10.B**

Justify the appropriateness of constructing a confidence interval for the difference between two population proportions by verifying conditions.

[Skill 4.E]

### ESSENTIAL KNOWLEDGE

**3.10.A.1**

Based on the sample data, a confidence interval can be calculated to estimate the difference between two population proportions. The appropriate confidence interval procedure is a two-sample z-interval for a difference between population proportions.

**3.10.A.2**

The parameters of a confidence interval for the difference between two population proportions should refer to the difference in the proportions, the response variable, and the populations in context.

**3.10.B.1**

A two-sample z-interval for a difference between two population proportions requires that three conditions be met:

**3.10.B.1.i**

The randomization condition—the data should be collected using two independent random samples or a randomized experiment.

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**LEARNING OBJECTIVE**

**3.10.B**

Justify the appropriateness of constructing a confidence interval for the difference between two population proportions by verifying conditions.

[Skill 4.E]

**3.10.C**

Calculate an appropriate confidence interval for the difference between two population proportions.

[Skill 3.E]

**3.10.D**

Calculate the standard error and margin of error for estimating the difference between two population proportions.

[Skill 3.E]

**ESSENTIAL KNOWLEDGE**

**3.10.B.1.ii**

The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size:  $n_1 \leq 10\%N_1$  and  $n_2 \leq 10\%N_2$ , where  $N_1$  is the size of population 1 and  $N_2$  is the size of population 2. The sample sizes are represented as  $n_1$  and  $n_2$ . (Note: This condition is unnecessary when the data are from a randomized experiment.)

**3.10.B.1.iii**

The normality condition—the number of observed successes,  $n_1\hat{p}_1$  and  $n_2\hat{p}_2$ , and number of observed failures,  $n_1(1-\hat{p}_1)$  and  $n_2(1-\hat{p}_2)$ , for both samples are all at least 10.

**3.10.C.1**

The point estimate for the difference between two population proportions is  $(\hat{p}_1 - \hat{p}_2)$ .

**3.10.C.2**

For the difference between two population proportions, the interval estimate can be constructed as point estimate  $\pm$  (margin of error). The interval estimate for the difference between two population proportions is

$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

**3.10.D.1**

The standard error (SE) for the difference between two population proportions is

$$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

**3.10.D.2**

For the difference between two population proportions, the margin of error is the critical value ( $z^*$ ) times the standard error (SE) of the difference between the two proportions, which

equals  $z^* \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ .

## SKILLS

## 4.F

Interpret results of statistical inference methods.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 3.11

# Justifying a Claim Based on a Confidence Interval for the Difference Between Two Population Proportions

## Required Course Content

### LEARNING OBJECTIVE

**3.11.A**

Interpret a confidence interval in context for the difference between two population proportions.

[Skill 4.F]

### ESSENTIAL KNOWLEDGE

**3.11.A.1**

Because the confidence interval for the difference between two population proportions is calculated based on samples from two populations, the computed interval may or may not contain the value for the difference between those two population proportions.

**3.11.A.2**

The interpretation of the confidence level is as follows: In repeated random sampling with the same sample sizes from the same populations, approximately  $C\%$  of confidence intervals created will capture the difference between the two population proportions, where  $C$  represents the numerical value of the confidence level used.

**3.11.A.3**

When interpreting a  $C\%$  confidence interval for the difference between two population proportions, we say we are  $C\%$  confident that the interval  $(a, b)$  contains the parameter for the difference between the populations, where  $a$  represents the lower limit and  $b$  represents the upper limit. An interpretation of a confidence interval for the difference between two population proportions includes a reference to the parameter with the details about the populations it represents in the context of the study.

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### LEARNING OBJECTIVE

**3.11.B**

Justify a claim based on a confidence interval for the difference between two population proportions.

**[Skill 4.G]**

### ESSENTIAL KNOWLEDGE

**3.11.B.1**

A confidence interval for the difference between two population proportions provides an interval of values that may provide convincing evidence to support a particular claim about the difference between the two population proportions. For example, if the interval contains 0, then there is insufficient evidence to conclude there is a difference between the two population proportions. If the interval does not contain 0, there is sufficient evidence to conclude there is a difference between the two population proportions.

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 2.E

Identify the null and alternative hypotheses.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 3.12

# Setting Up a Test for the Difference Between Two Population Proportions

## Required Course Content

### LEARNING OBJECTIVE

**3.12.A**

Identify an appropriate testing method for the difference between population proportions including the parameters.

[Skill 2.C]

**3.12.B**

Identify the null and alternative hypotheses for the difference between population proportions.

[Skill 2.E]

### ESSENTIAL KNOWLEDGE

**3.12.A.1**

The appropriate testing method for the difference between two population proportions is a two-sample z-test for the difference between two population proportions.

**3.12.A.2**

The parameters for a hypothesis test for the difference between two population proportions should reference the population parameters, the response variables, and the populations in context.

**3.12.B.1**

For a two-sample z-test for the difference between two population proportions, the null hypothesis indicates no difference. The null hypothesis for the difference between two population proportions can be written as either  $H_0: p_1 = p_2$  or  $H_0: p_1 - p_2 = 0$ . A one-sided alternative hypothesis for the difference between two population proportions can be written as either  $H_a: p_1 < p_2$  or equivalently  $H_a: p_1 - p_2 < 0$ , or  $H_a: p_1 > p_2$  or equivalently  $H_a: p_1 - p_2 > 0$ . A two-sided alternative hypothesis for the difference between two population proportions can be written as either  $H_a: p_1 \neq p_2$  or equivalently  $H_a: p_1 - p_2 \neq 0$ .

**LEARNING OBJECTIVE****3.12.C**

Justify the appropriateness of a hypothesis test for the difference between two population proportions by verifying conditions.

[Skill 4.E]

**ESSENTIAL KNOWLEDGE****3.12.C.1**

A two-sample z-test for a difference between two population proportions requires that three conditions be met:

**3.12.C.1.i**

The randomization condition—the data should be collected using two independent random samples or a randomized experiment.

**3.12.C.1.ii**

The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size:  $n_1 \leq 10\%N_1$  and  $n_2 \leq 10\%N_2$ , where  $N_1$  is the size of population 1 and  $N_2$  is the size of population 2. The sample sizes are represented as  $n_1$  and  $n_2$ . (Note: This condition is unnecessary when the data are from a randomized experiment.)

**3.12.C.1.iii**

The normality condition— $n_1\hat{p}_c$ ,  $n_1(1-\hat{p}_c)$ ,  $n_2\hat{p}_c$ , and  $n_2(1-\hat{p}_c)$ , must all be at least 10, with  $\hat{p}_c = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$  being the combined (or pooled) proportion assuming that  $H_0$  is true ( $H_0: p_1 = p_2$  or  $H_0: p_1 - p_2 = 0$ ).

SKILLS

**3.E**  
Calculate appropriate statistical inference method results.

**4.F**  
Interpret results of statistical inference methods.

**4.G**  
Justify a claim based on statistical inference method results.

TOPIC 3.13

# Carrying Out a Test for the Difference Between Two Population Proportions

## Required Course Content

### LEARNING OBJECTIVE

**3.13.A**

Calculate an appropriate test statistic and  $p$ -value for testing a hypothesis for the difference between two population proportions.

[Skill 3.E]

**3.13.B**

Interpret the  $p$ -value of a hypothesis test for the difference between two population proportions.

[Skill 4.F]

### ESSENTIAL KNOWLEDGE

**3.13.A.1**

The test statistic for the difference between two population proportions is

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}_c(1 - \hat{p}_c) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}, \text{ where}$$

$$\hat{p}_c = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2} \text{ is the proportion of}$$

successes for the two groups combined. The  $z$ -statistic has a standard normal distribution when the null hypothesis is true.

**3.13.A.2**

The  $p$ -value for a two-sample  $z$ -test for the difference between two population proportions can be found from the standard normal distribution using a table or technology.

**3.13.B.1**

The  $p$ -value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the  $p$ -value of a hypothesis test for a difference between two population proportions should include a statement that the  $p$ -value is computed assuming the null hypothesis is true (i.e., by assuming that the true population proportions are equal to each other in context).

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**LEARNING OBJECTIVE****3.13.C**

Justify a claim about the populations based on the results of a hypothesis test for the difference between two population proportions.

**[Skill 4.G]****ESSENTIAL KNOWLEDGE****3.13.C.1**

A formal decision in a hypothesis test for the difference between two population proportions explicitly compares the  $p$ -value to the significance level,  $\alpha$ . If the  $p$ -value  $\leq \alpha$ , then reject the null hypothesis,  $H_0: p_1 = p_2$  or  $H_0: p_1 - p_2 = 0$ . If the  $p$ -value  $> \alpha$ , then fail to reject the null hypothesis.

**3.13.C.2**

The results of a hypothesis test for the difference between two population proportions can serve as the statistical reasoning to support the answer to an investigative question about the two populations that were sampled.

**3.13.C.3**

A conclusion for the hypothesis test for the difference between two population proportions is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the parameters and the populations.

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 2.E

Identify the null and alternative hypotheses.

## 4.C

Describe distributions and compare relative positions of points within a distribution.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 3.14

# Setting Up a Chi-Square Test for Homogeneity or Independence

## Required Course Content

### LEARNING OBJECTIVE

**3.14.A**

Describe chi-square distributions.

[Skill 4.C]

**3.14.B**

Identify an appropriate testing method for comparing distributions in two-way tables of categorical data including the populations and variables.

[Skill 2.C]

### ESSENTIAL KNOWLEDGE

**3.14.A.1**

The chi-square statistic measures the distance between observed and expected counts relative to expected counts.

**3.14.A.2**

Chi-square distributions have positive values and are skewed right. Within this family of density curves, the skew becomes less pronounced with increasing degrees of freedom.

**3.14.B.1**

To determine whether the distributions of a categorical variable for two or more populations are different, the appropriate test is the chi-square test for homogeneity.

**3.14.B.2**

A chi-square test for homogeneity should reference the categorical variable and the populations in context.

**3.14.B.3**

To determine whether row and column variables in a two-way table of categorical data might be associated in the single population from which the data were sampled, the appropriate test is the chi-square test for independence.

**3.14.B.4**

A chi-square test for independence should reference the categorical variables and the population in context.

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**LEARNING OBJECTIVE****3.14.C**

Identify the null and alternative hypotheses for a chi-square test for homogeneity or independence.

**[Skill 2.E]**

**3.14.D**

Justify the appropriateness of a chi-square test for independence or homogeneity by verifying conditions.

**[Skill 4.E]**

**ESSENTIAL KNOWLEDGE****3.14.C.1**

The appropriate null hypothesis for a chi-square test for homogeneity is  $H_0$ : there is no difference in the distributions of the categorical variable across populations or treatments. The appropriate alternative hypothesis for a chi-square test for homogeneity is  $H_a$ : there is a difference in the distributions of the categorical variable across populations or treatments.

**3.14.C.2**

The appropriate null hypothesis for a chi-square test for independence is  $H_0$ : there is no association between two categorical variables in a given population or the two categorical variables in a given population are independent of each other. The appropriate alternative hypothesis for a chi-square test for independence is  $H_a$ : there is an association between two categorical variables in a given population or the two categorical variables in a given population are not independent of each other.

**3.14.D.1**

A chi-square test for homogeneity or independence requires that three conditions must be met:

**3.14.D.1.i**

The randomization condition—the test of independence states that the data should be collected using a random sample. The test for homogeneity states that the data should be collected using independent random samples or a randomized experiment.

**3.14.D.1.ii**

The 10% condition—when sampling without replacement, check that  $n \leq 10\%N$ , where  $N$  is the size of the population and  $n$  is the sample size. (Note: This condition is unnecessary when the data are from a randomized experiment.)

**3.14.D.1.iii**

The expected counts condition—all expected counts should be greater than 5.

SKILLS

3.C

Calculate and estimate expected counts, percentages, probabilities, and intervals.

3.E

Calculate appropriate statistical inference method results.

4.F

Interpret results of statistical inference methods.

4.G

Justify a claim based on statistical inference method results.

TOPIC 3.15

# Carrying Out a Chi-Square Test for Homogeneity or Independence

## Required Course Content

### LEARNING OBJECTIVE

3.15.A

Calculate expected counts for two-way tables of categorical data.

[Skill 3.C]

3.15.B

Calculate the appropriate test statistic and  $p$ -value for a chi-square test for homogeneity or independence.

[Skill 3.E]

### ESSENTIAL KNOWLEDGE

3.15.A.1

The expected counts (under the null hypothesis) in a particular cell of a two-way table of categorical data can be calculated using the formula

$$\text{expected count} = \frac{(\text{row total})(\text{column total})}{\text{total table}}$$

3.15.B.1

The appropriate test statistic for a chi-square test for homogeneity or independence is the chi-square statistic

$$\chi^2 = \sum \frac{(\text{Observed Count} - \text{Expected Count})^2}{\text{Expected Count}},$$

where the sum is taken over all cells of the two-way table. The chi-square statistics have a chi-square distribution with degrees of freedom equal to  $(\text{number of rows} - 1) \cdot (\text{number of columns} - 1)$  when the null hypothesis is true.

3.15.B.2

The  $p$ -value for a chi-square test for independence or homogeneity is found from a chi-square distribution using a table or technology.

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**LEARNING OBJECTIVE**

**3.15.C**

Interpret the  $p$ -value for the chi-square test for homogeneity or independence.

**[Skill 4.F]**

**3.15.D**

Justify a claim about the population(s) based on the results of a chi-square test for homogeneity or independence.

**[Skill 4.G]**

**ESSENTIAL KNOWLEDGE**

**3.15.C.1**

The  $p$ -value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the  $p$ -value for the chi-square test for homogeneity or independence should include a statement that the  $p$ -value is computed by assuming the null hypothesis is true in context.

**3.15.D.1**

A formal decision in a hypothesis test explicitly compares the  $p$ -value to the significance level,  $\alpha$ . If the  $p$ -value  $\leq \alpha$ , then reject the null hypothesis for the appropriate chi-square test. If the  $p$ -value  $> \alpha$ , then fail to reject the null hypothesis.

**3.15.D.2**

The results of a chi-square test for homogeneity or independence can serve as the statistical reasoning to support the answer to an investigative question about the population that was sampled (independence) or the populations that were sampled (homogeneity).

**3.15.D.3**

A conclusion for a chi-square test for homogeneity or independence is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the population(s).

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## AP STATISTICS

### UNIT 4

# Inference for Quantitative Data: Means



**10–20%**  
AP EXAM WEIGHTING



**~18**  
CLASS PERIODS



Remember to go to [AP Classroom](#) to assign students the online **Progress Checks** for this unit.

Whether assigned as homework or completed in class, the **Progress Checks** provide each student with immediate feedback related to this unit's topics and skills.

#### **Progress Check 4**

**Multiple-choice: ~49 questions**

**Free-response: 3 questions**

- Question 3: Inference
- Question 3: Inference
- Question 3: Inference

# Inference for Quantitative Data: Means

## ESSENTIAL QUESTIONS

- How can we compare the average salaries of different professions across different regions of a country?
- How can we evaluate the effectiveness of a new medicine in lowering cholesterol for older adults?
- How can a pre-test and a post-test be used to evaluate student learning?



## Developing Understanding

In this unit, students analyze quantitative data to make inferences about population means. Students learn that  $t^*$  and  $t$ -tests are used for inference with means when the population standard deviation,  $\sigma$ , is unknown. Using  $s$  for  $\sigma$  in the formula for  $z$  gives a slightly different value,  $t$ . The distribution of  $t$  depends on sample size and has more area in the tails than a normal distribution. The boundaries for rejecting a null hypothesis using a  $t$ -distribution tend to be further from the mean than for a normal distribution. Students discover how and why conditions for inference with proportions and means are similar and different.

## Building Statistical Practices

2.C 2.D 2.E 3.D 3.E 4.C 4.D 4.E 4.F 4.G

Unit 4 focuses on means, which have similarities to the conditions and procedures for proportions. Because students sometimes confuse  $t$ -tests with  $z$ -tests, teachers might consider carving out time to review the underlying rationales each time conditions are discussed. These reviews help students develop understanding through repeated practice in new situations. Teachers can encourage students to be mindful of notation and reinforce that inference testing requires careful selection of a procedure based on specific conditions for a given problem. Common student errors include mislabeling conditions (e.g., incorrectly associating the 10% condition with the randomization condition), relying on vague references to the normal distribution, or applying the sample data condition incorrectly. The null and alternative hypotheses must be clearly stated in terms of population parameters, not sample statistics. A formal decision compares the  $p$ -value to the level of significance. Students will benefit from having opportunities to practice providing a numerical reference to support their claim (e.g., we reject the null hypothesis because  $p\text{-value} \leq 0.05$ ) and interpret findings within the context of the question.

## Preparing for the AP Exam

These concepts will be evaluated using individual multiple-choice questions and parts of free-response questions. Students are expected to recognize that free-response questions asking whether data provide convincing evidence of some findings are asking for a significance test, not just a descriptive analysis. When using statistical inference for hypothesis tests, students need to identify the correct parameter and hypotheses, identify an appropriate test procedure and check conditions, calculate a test statistic and  $p$ -value, and provide a conclusion in context, along with a justification based on linkage between the  $p$ -value and the level of significance. Students will benefit from having opportunities to practice inference procedures from Units 3 and 4 so they understand the differences and similarities.

## UNIT AT A GLANCE

Topic	Skills	Class Periods ~18 CLASS PERIODS
<b>4.1 Sampling Distributions for Sample Means</b>	<p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	1
<b>4.2 Constructing a Confidence Interval for a Population Mean or Population Mean Difference</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.C</b> Describe distributions and compare relative positions of points within a distribution.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>4.3 Justifying a Claim Based on a Confidence Interval for a Population Mean or Population Mean Difference</b>	<p><b>2.D</b> Identify types of errors and relationships among components in statistical inference methods.</p> <p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>4.4 Setting Up a Test for a Population Mean or Population Mean Difference</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>2.E</b> Identify the null and alternative hypotheses.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>4.5 Carrying Out a Test for a Population Mean or Population Mean Difference</b>	<p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>4.6 Sampling Distributions for the Difference Between Two Sample Means</b>	<p><b>3.D</b> Calculate means, standard deviations, and parameters for probability distributions.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	1

(Continued)

## UNIT AT A GLANCE (cont'd)

Topic	Skills	Class Periods ~18 CLASS PERIODS
<b>4.7 Constructing a Confidence Interval for the Difference Between Two Population Means</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>4.8 Justifying a Claim Based on a Confidence Interval for the Difference Between Two Population Means</b>	<p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
<b>4.9 Setting Up a Test for the Difference Between Two Population Means</b>	<p><b>2.C</b> Identify appropriate statistical inference methods.</p> <p><b>2.E</b> Identify the null and alternative hypotheses.</p> <p><b>4.E</b> Justify the use of a chosen statistical inference method by verifying conditions.</p>	2
<b>4.10 Carrying Out a Test for the Difference Between Two Population Means</b>	<p><b>3.E</b> Calculate appropriate statistical inference method results.</p> <p><b>4.F</b> Interpret results of statistical inference methods.</p> <p><b>4.G</b> Justify a claim based on statistical inference method results.</p>	2
 Go to <a href="#">AP Classroom</a> to assign the <b>Progress Check</b> for Unit 4. Review the results in class to identify and address any student misunderstandings.		

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit.

Activity	Topics	Sample Activity
1	4.1	<p><b>Quick Write</b></p> <p>Provide students with a prompt such as "Describe how to construct a sampling distribution." Then allow 1 or 2 minutes for them to write a response (e.g., "obtain sample, collect responses, calculate statistic, plot statistic, repeat"). Allow more writing time for more difficult prompts.</p>
2	4.2	<p><b>Discussion Groups</b></p> <p>Place students in groups of three or four and ask each group to identify the conditions for performing a test about a population mean. For each condition, have them explain why the condition is required and what would go wrong with the test if the condition were violated. Have groups pair up and compare answers.</p>
3	4.2 4.4 4.7 4.9	<p><b>Team Challenge: Marking the Text</b></p> <p>Give students examples of statistical situations. Have them highlight or underline the key phrases to help them determine which statistical test to use for each situation. For example, does it require a significance test or a confidence interval? Does it involve means or proportions? Is there only one variable, or are there two or more variables?</p>
4	4.2 4.3 4.4 4.5 4.7 4.8 4.9 4.10	<p><b>Round Table</b></p> <p>Arrange students in groups of four and give each student an identical paper containing four different inference procedure problems from Unit 4. Have students complete the first one on their paper and then pass the paper clockwise to another member in their group. That student checks the first problem and then completes the second problem on the paper. Have students rotate again, continuing the process until each student has their original paper back.</p>
5	4.2 4.4 4.7 4.9	<p><b>Graphic Organizer</b></p> <p>Have students work in teams of two or three to develop a flowchart for determining which inference procedure from Units 3 and 4 to use in a given setting.</p>

## TOPIC 4.1

# Sampling Distributions for Sample Means

## Required Course Content

### LEARNING OBJECTIVE

**4.1.A**

Calculate the mean and standard deviation of a sampling distribution of a sample mean.

[Skill 3.D]

**4.1.B**

Justify the appropriateness of conditions for the sampling distribution of a sample mean.

[Skill 4.E]

### ESSENTIAL KNOWLEDGE

**4.1.A.1**

For a population with population mean  $\mu$  and population standard deviation  $\sigma$ , when the sampled values are independent, the sampling distribution of the sample mean has mean

$$\mu_{\bar{x}} = \mu \text{ and standard deviation } \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

**4.1.B.1**

Sampling without replacement requires that two conditions must be met:

**4.1.B.1.i**

The randomization condition—the data should be collected using a random sample.

**4.1.B.1.ii**

The 10% condition—the population size must be at least 10 times larger than the sample size ( $n \leq 10\%N$ ), where  $N$  is the size of the population and  $n$  is the sample size.

**4.1.B.2**

For a quantitative variable, if the population distribution can be modeled by a normal distribution, the sampling distribution of the sample mean,  $\bar{x}$ , can be modeled with a normal distribution regardless of the sample size.

**SKILLS**
**3.D**

Calculate means, standard deviations, and parameters for probability distributions.

**4.D**

Interpret statistical calculations and results to assess meaning or a claim.

**4.E**

Justify the use of a chosen statistical inference method by verifying conditions.

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## LEARNING OBJECTIVE

**4.1.B**

Justify the appropriateness of conditions for the sampling distribution of a sample mean.

[Skill 4.E]

**4.1.C**

Interpret the mean, standard deviation, and probabilities for the sampling distribution of a sample mean.

[Skill 4.D]

## ESSENTIAL KNOWLEDGE

**4.1.B.3**

For a quantitative variable, if the population distribution cannot be modeled by a normal distribution, the sampling distribution of the sample mean,  $\bar{x}$ , can be modeled approximately by a normal distribution, provided  $n \geq 30$ . If the population distribution is extremely skewed, a sample size much larger than 30 may be needed to ensure the sampling distribution is approximately normal.

**4.1.C.1**

The mean, standard deviation, and probabilities for a sampling distribution of a sample mean should be interpreted within the context of a specific population.

## TOPIC 4.2

# Constructing a Confidence Interval for a Population Mean or Population Mean Difference

## Required Course Content

### LEARNING OBJECTIVE

**4.2.A**

Describe  $t$ -distributions.

[Skill 4.C]

**4.2.B**

Identify an appropriate confidence interval procedure including the parameter for a population mean or population mean difference.

[Skill 2.C]

### ESSENTIAL KNOWLEDGE

**4.2.A.1**

$t$ -distributions, also called Student's  $t$ -distributions, form a family of symmetric, bell-shaped, standardized distributions with wider tails than that of the standard normal distribution. Specific  $t$ -distributions are identified using a parameter known as the number of degrees of freedom ( $df$ ), which is based on the sample size(s). When the degrees of freedom are small, the  $t$ -distribution has a much narrower peak and fatter tails than a normal distribution. As the degrees of freedom increase, the  $t$ -distribution more closely resembles the standard normal distribution (mean  $\mu = 0$  and standard deviation  $\sigma = 1$ ).

**4.2.A.2**

$t$ -distributions are used for finding critical values and test statistics for inferences about a population mean,  $\mu$ , when the population standard deviation,  $\sigma$ , is unknown and the sample standard deviation,  $s$ , must be used instead.

**4.2.B.1**

The appropriate confidence interval procedure for estimating the population mean of a quantitative variable for one sample is a one-sample  $t$ -interval for a population mean. (The population standard deviation,  $\sigma$ , is not typically known for distributions for quantitative variables.)

### SKILLS

**2.C**

Identify appropriate statistical inference methods.

**3.E**

Calculate appropriate statistical inference method results.

**4.C**

Describe distributions and compare relative positions of points within a distribution.

**4.E**

Justify the use of a chosen statistical inference method by verifying conditions.

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## LEARNING OBJECTIVE

## 4.2.B

Identify an appropriate confidence interval procedure including the parameter for a population mean or population mean difference.

[Skill 2.C]

## 4.2.C

Justify the appropriateness of constructing a confidence interval for a population mean or population mean difference by verifying conditions.

[Skill 4.E]

## ESSENTIAL KNOWLEDGE

## 4.2.B.2

For a matched pairs design with two dependent samples, the appropriate analysis calculates differences between pairs of values to produce one sample of differences. The confidence interval procedure for the matched pairs design is a one-sample  $t$ -interval for a population mean difference.

## 4.2.B.3

The parameter for a confidence interval for a population mean or population mean difference should reference the population mean or population mean difference and the response variable, in context. For the population mean difference, it is important to state the order of subtraction for the difference.

## 4.2.C.1

A one-sample  $t$ -interval for a population mean or population mean difference requires that three conditions be met:

## 4.2.C.1.i

The randomization condition—the data should be collected using a random sample or a randomized experiment.

## 4.2.C.1.ii

The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size ( $n \leq 10\%N$ ), where  $N$  is the size of the population and  $n$  is the sample size.

## 4.2.C.1.iii

The sample data condition—it is indicated the population distribution is approximately normal, or  $n \geq 30$ , or if  $n < 30$ , the sample data distribution should be free from strong skewness and outliers. For matched pairs, the number of differences should be greater than or equal to 30. If the number of differences is less than 30, the sample of differences should be free from strong skewness and outliers.

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**LEARNING OBJECTIVE**

**4.2.D**

Calculate an appropriate confidence interval for a population mean or population mean difference.

[Skill 3.E]

**4.2.E**

Calculate the standard error and margin of error for a sample size for a one-sample  $t$ -interval.

[Skill 3.E]

**ESSENTIAL KNOWLEDGE**

**4.2.D.1**

A point estimate for a population mean is the sample mean,  $\bar{x}$ , or  $\bar{x}_d$  for the sample mean difference.

**4.2.D.2**

To estimate the population mean for one sample or the population mean difference between values in matched pairs, when the population standard deviation is unknown, the confidence interval is  $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$ , where  $t^*$  is the critical value for the central  $C\%$  of a  $t$ -distribution with degrees of freedom  $n - 1$ .

**4.2.E.1**

The standard error ( $SE$ ) for a sample mean is given by  $SE_{\bar{x}} = \frac{s}{\sqrt{n}}$ .

**4.2.E.2**

For a one-sample  $t$ -interval for a population mean, the margin of error is the critical value ( $t^*$ ) times the standard error ( $SE$ ), which equals  $(t^*) \left( \frac{s}{\sqrt{n}} \right)$ .

## SKILLS

## 2.D

Identify types of errors and relationships among components in statistical inference methods.

## 4.F

Interpret results of statistical inference methods.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 4.3

# Justifying a Claim Based on a Confidence Interval for a Population Mean or Population Mean Difference

## Required Course Content

### LEARNING OBJECTIVE

## 4.3.A

Interpret a confidence interval in context for a population mean or population mean difference.

[Skill 4.F]

### ESSENTIAL KNOWLEDGE

## 4.3.A.1

Because the confidence interval for a population mean or population mean difference is calculated based on a sample from a population, the computed interval may or may not contain the value of the population mean or population mean difference.

## 4.3.A.2

The interpretation of the confidence level is as follows: In repeated random sampling with the same sample size from the same population, approximately  $C\%$  of confidence intervals created will capture the population mean or population mean difference, where  $C$  represents the numerical value of the confidence level used.

## 4.3.A.3

When interpreting a  $C\%$  confidence interval for a population mean or population mean difference, we say we are  $C\%$  confident the interval  $(a, b)$  contains the value of the population mean or population mean difference, where  $a$  represents the lower limit and  $b$  represents the upper limit. An interpretation of a confidence interval for a population mean or population mean difference includes a reference to the parameter.

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**LEARNING OBJECTIVE****4.3.B**

Justify a claim based on a confidence interval for a population mean or population mean difference.

[Skill 4.G]

**4.3.C**

Identify the relationships among sample size, confidence interval width, confidence level, and margin of error for a population mean or population mean difference.

[Skill 2.D]

**ESSENTIAL KNOWLEDGE****4.3.B.1**

A confidence interval for a population mean or population mean difference provides an interval of values that may serve as convincing evidence to support a particular claim about the population mean or population mean difference.

**4.3.C.1**

For a given sample, increasing the confidence level will result in the following:

**4.3.C.1.i**

The critical value will increase.

**4.3.C.1.ii**

The margin of error will increase.

**4.3.C.1.iii**

The width of the confidence interval will increase.

**4.3.C.2**

Increasing the sample size decreases the standard error. Thus, when all other things remain the same, the width of a confidence interval for a population mean or population mean difference tends to decrease as the sample size increases. For a confidence interval for a population mean or population mean difference with a given confidence level, the width of the interval is approximately

proportional to  $\frac{1}{\sqrt{n}}$ .

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 2.E

Identify the null and alternative hypotheses.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 4.4

# Setting Up a Test for a Population Mean or Population Mean Difference

## Required Course Content

### LEARNING OBJECTIVE

## 4.4.A

Identify an appropriate testing method and parameter for a population mean or population mean difference with unknown  $\sigma$ .

[Skill 2.C]

## 4.4.B

Identify the null and alternative hypotheses for a population mean or population mean difference with unknown  $\sigma$ .

[Skill 2.E]

### ESSENTIAL KNOWLEDGE

## 4.4.A.1

The appropriate test for a population mean with unknown population standard deviation  $\sigma$  is a one-sample  $t$ -test for a population mean.

## 4.4.A.2

For a matched pairs design with two dependent samples, the appropriate analysis calculates differences between pairs of values to produce one sample of differences. The hypothesis testing procedure for the matched pairs design is a one-sample  $t$ -test for the population mean difference.

## 4.4.A.3

The parameter for a hypothesis test for a population mean and population mean difference should reference the population parameter, the response variable, and the population in context.

## 4.4.B.1

The null hypothesis for a one-sample  $t$ -test for a population mean is  $H_0: \mu = \mu_0$ , in which  $\mu_0$  is the null hypothesized value for the population mean. A one-sided alternative hypothesis for a one-sample  $t$ -test for a population mean is either  $H_a: \mu < \mu_0$  or  $H_a: \mu > \mu_0$ . A two-sided alternative hypothesis is  $H_a: \mu \neq \mu_0$ .

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## LEARNING OBJECTIVE

**4.4.B**

Identify the null and alternative hypotheses for a population mean or population mean difference with unknown  $\sigma$ .

[Skill 2.E]

**4.4.C**

Justify the appropriateness of a hypothesis test for a population mean or population mean difference by verifying conditions.

[Skill 4.E]

## ESSENTIAL KNOWLEDGE

**4.4.B.2**

The null hypothesis for a population mean difference is  $H_0: \mu_d = 0$ . A one-sided alternative hypothesis for a population mean difference is either  $H_a: \mu_d < 0$  or  $H_a: \mu_d > 0$ . A two-sided alternative hypothesis is  $H_a: \mu_d \neq 0$ .

**4.4.C.1**

A one-sample  $t$ -test for a population mean or a population mean difference requires that three conditions be met:

**4.4.C.1.i**

The randomization condition—the data should be collected using a random sample or a randomized experiment.

**4.4.C.1.ii**

The 10% condition—when sampling without replacement, the population size must be at least 10 times larger than the sample size ( $n \leq 10\%N$ ), where  $N$  is the size of the population and  $n$  is the sample size.

**4.4.C.1.iii**

The sample data condition—it is indicated the population distribution is approximately normal, or  $n \geq 30$ , or if  $n < 30$ , the sample data distribution should be free from strong skewness and outliers. For matched pairs, the number of differences should be greater than or equal to 30. If the number of differences is less than 30, the sample of differences should be free from strong skewness and outliers.

## SKILLS

## 3.E

Calculate appropriate statistical inference method results.

## 4.F

Interpret results of statistical inference methods.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 4.5

# Carrying Out a Test for a Population Mean or Population Mean Difference

## Required Course Content

### LEARNING OBJECTIVE

## 4.5.A

Calculate an appropriate test statistic and  $p$ -value for testing a hypothesis about a population mean or population mean difference.

[Skill 3.E]

## 4.5.B

Interpret the  $p$ -value of a hypothesis test for a population mean or population mean difference.

[Skill 4.F]

### ESSENTIAL KNOWLEDGE

## 4.5.A.1

The test statistic for a one-sample  $t$ -test for a population mean or population mean

difference is  $t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ , where  $t$  has degrees

of freedom  $n - 1$ . The  $t$ -statistic has a  $t$ -distribution with degrees of freedom  $n - 1$  when the null hypothesis is true.

## 4.5.A.2

The  $p$ -value for a one-sample  $t$ -test for a population mean or population mean difference is found using the appropriate  $t$ -distribution table or technology.

## 4.5.B.1

The  $p$ -value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the  $p$ -value of a hypothesis test for a population mean or population mean difference should include a statement that the  $p$ -value is computed by assuming that the null hypothesis is true (i.e., by assuming that the population mean is equal to the particular value stated in the null hypothesis in context).

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**LEARNING OBJECTIVE****4.5.C**

Justify a claim about the population based on the results of a hypothesis test for a population mean or population mean difference.

**[Skill 4.G]****ESSENTIAL KNOWLEDGE****4.5.C.1**

A formal decision explicitly compares the  $p$ -value to the significance level,  $\alpha$ . If the  $p$ -value  $\leq \alpha$ , then reject the null hypothesis,  $H_0: \mu = \mu_0$ . If the  $p$ -value  $> \alpha$ , then fail to reject the null hypothesis.

**4.5.C.2**

The results of a hypothesis test for a population mean or population mean difference can serve as the statistical reasoning to support the answer to an investigative question about the population that was sampled.

**4.5.C.3**

A conclusion for the hypothesis test for a population mean or population mean difference is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the parameter and the population.

## SKILLS

## 3.D

Calculate means, standard deviations, and parameters for probability distributions.

## 4.D

Interpret statistical calculations and results to assess meaning or a claim.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 4.6

# Sampling Distributions for the Difference Between Two Sample Means

## Required Course Content

### LEARNING OBJECTIVE

## 4.6.A

Calculate the mean and standard deviation of a sampling distribution for the difference between two sample means.

[Skill 3.D]

## 4.6.B

Justify the appropriateness of conditions for the sampling distribution of the difference between two sample means.

[Skill 4.E]

### ESSENTIAL KNOWLEDGE

## 4.6.A.1

For two independent populations with population means  $\mu_1$  and  $\mu_2$  and population standard deviations  $\sigma_1$  and  $\sigma_2$ , when the sampled values are independent, the sampling distribution of the difference in sample means  $\bar{x}_1 - \bar{x}_2$  has a mean  $\mu_{(\bar{x}_1 - \bar{x}_2)} = \mu_1 - \mu_2$  and

standard deviation  $\sigma_{(\bar{x}_1 - \bar{x}_2)} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ .

## 4.6.B.1

Sampling without replacement requires that two conditions must be met:

## 4.6.B.1.i

The randomization condition—the data should be collected using two independent random samples.

## 4.6.B.1.ii

The 10% condition—the size of each sample should be less than or equal to 10% of the respective population size:  $n_1 \leq 10\%N_1$  and  $n_2 \leq 10\%N_2$ , where  $N_1$  is the size of population 1 and  $N_2$  is the size of population 2. The sample sizes are represented as  $n_1$  and  $n_2$ .

## 4.6.B.2

If the data come from an experiment, the data only need to meet the randomization condition. The treatments must be randomly assigned to experimental units to meet the randomization condition.

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**LEARNING OBJECTIVE**

**4.6.B**

Justify the appropriateness of conditions for the sampling distribution of the difference between two sample means.

[Skill 4.E]

**4.6.C**

Interpret the mean, standard deviation, and probabilities for a sampling distribution for the difference between sample means.

[Skill 4.D]

**ESSENTIAL KNOWLEDGE**

**4.6.B.3**

The sampling distribution for the difference between sample means,  $\bar{x}_1 - \bar{x}_2$ , can be modeled with a normal distribution if the two population distributions can each be modeled by a normal distribution.

**4.6.B.4**

The sampling distribution for the difference between sample means,  $\bar{x}_1 - \bar{x}_2$ , can be modeled approximately by a normal distribution if the two population distributions cannot be modeled by a normal distribution but  $n_1 \geq 30$  and  $n_2 \geq 30$ .

**4.6.C.1**

The mean, standard deviation, and probabilities for a sampling distribution for the difference between sample means should be interpreted within the context of specific populations.

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 3.E

Calculate appropriate statistical inference method results.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 4.7

# Constructing a Confidence Interval for the Difference Between Two Population Means

## Required Course Content

### LEARNING OBJECTIVE

## 4.7.A

Identify an appropriate confidence interval procedure including the parameter for the difference between two population means.

[Skill 2.C]

## 4.7.B

Justify the appropriateness of constructing a confidence interval for the difference between two population means by verifying conditions.

[Skill 4.E]

### ESSENTIAL KNOWLEDGE

## 4.7.A.1

Based on the sample data, a confidence interval can be calculated to estimate the difference between two population means. The appropriate confidence interval procedure for two independent samples is a two-sample  $t$ -interval for the difference between population means.

## 4.7.A.2

The parameter for a confidence interval for a two-sample  $t$ -interval for the difference between population means should reference the difference in the means, the response variable, and the populations in context.

## 4.7.B.1

A two-sample  $t$ -interval for a difference between population means requires that three conditions be met:

## 4.7.B.1.i

The randomization condition—the data should be collected using two independent random samples or a randomized experiment.

## 4.7.B.1.ii

The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size:  $n_1 \leq 10\%N_1$  and  $n_2 \leq 10\%N_2$ , where  $N_1$  is the size of population 1 and  $N_2$  is the size of population 2. The sample sizes are represented as  $n_1$  and  $n_2$ . (Note: This condition is unnecessary when the data are from a randomized experiment).

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**LEARNING OBJECTIVE**

**4.7.B**

Justify the appropriateness of constructing a confidence interval for the difference between two population means by verifying conditions.

[Skill 4.E]

**4.7.C**

Calculate an appropriate confidence interval for the difference between two population means.

[Skill 3.E]

**4.7.D**

Calculate the standard error and margin of error for estimating the difference between two population means.

[Skill 3.E]

**ESSENTIAL KNOWLEDGE**

**4.7.B.1.iii**

The sample data condition—both samples should have a sample size greater than or equal to 30 or it is indicated that both population distributions are approximately normal. If either sample size is less than 30, both sample data distributions should be free from strong skewness and outliers.

**4.7.C.1**

A point estimate for the difference between two population means is the difference in sample means,  $\bar{x}_1 - \bar{x}_2$ .

**4.7.C.2**

For the difference between population means when the population standard deviations are unknown, the confidence interval can be constructed as point estimate  $\pm$  (margin of error). The confidence interval for the difference between

population means is  $(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ ,

where  $t^*$  are the critical values for the central C% of a  $t$ -distribution with appropriate degrees of freedom that can be found using technology. The degrees of freedom fall between  $n_1 + n_2 - 2$  and the smaller of  $n_1 - 1$  and  $n_2 - 1$ .

**4.7.D.1**

The standard error for the difference between two sample means is  $SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ , where  $s_1$  and  $s_2$  are the sample standard deviations.

**4.7.D.2**

For the difference between two sample means, the margin of error is the critical value ( $t^*$ ) times the standard error (SE) of the difference of two sample means, which equals  $t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ .

## SKILLS

## 4.F

Interpret results of statistical inference methods.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 4.8

# Justifying a Claim Based on a Confidence Interval for the Difference Between Two Population Means

## Required Course Content

### LEARNING OBJECTIVE

## 4.8.A

Interpret a confidence interval in context for the difference between two population means.

[Skill 4.F]

### ESSENTIAL KNOWLEDGE

## 4.8.A.1

Because the confidence interval for the difference between two population means is calculated based on samples from two populations, the computed interval may or may not contain the value for the difference between the two population means.

## 4.8.A.2

The interpretation of the confidence level is as follows: In repeated random sampling with the same sample size from the same populations, approximately  $C\%$  of confidence intervals created will capture the difference between the two population means, where  $C$  represents the numerical value of the confidence level used.

## 4.8.A.3

When interpreting a  $C\%$  confidence interval for the difference between two population means, we say we are  $C\%$  confident that the interval  $(a, b)$  contains the value of the difference in the population means, where  $a$  represents the lower limit and  $b$  represents the upper limit. An interpretation of a confidence interval for the difference between two population means includes a reference to the difference in the population means with the details about the populations it represents in the context of the study.

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## LEARNING OBJECTIVE

### 4.8.B

Justify a claim based on a confidence interval for the difference between two population means.

**[Skill 4.G]**

## ESSENTIAL KNOWLEDGE

### 4.8.B.1

A confidence interval for the difference between two population means provides an interval of values that may serve as convincing evidence to support a particular claim about the difference in two population means. For example, if the interval contains 0, then there is insufficient evidence to conclude there is a difference between the two population means. If the interval does not contain 0, then there is sufficient evidence to conclude there is a difference between the two population means.

## SKILLS

## 2.C

Identify appropriate statistical inference methods.

## 2.E

Identify the null and alternative hypotheses.

## 4.E

Justify the use of a chosen statistical inference method by verifying conditions.

## TOPIC 4.9

# Setting Up a Test for the Difference Between Two Population Means

## Required Course Content

### LEARNING OBJECTIVE

## 4.9.A

Identify an appropriate testing method for the difference between two population means including the parameters for the difference between the two population means.

[Skill 2.C]

## 4.9.B

Identify the null and alternative hypotheses for the difference between two population means.

[Skill 2.E]

### ESSENTIAL KNOWLEDGE

## 4.9.A.1

The appropriate test for the difference between two population means is a two-sample  $t$ -test for a difference between two population means.

## 4.9.A.2

The parameters for a hypothesis test for the difference between two population means should reference the population parameters, the response variables, and the populations in context.

## 4.9.B.1

The null hypothesis for a two-sample  $t$ -test for the difference between two population means,  $\mu_1$  and  $\mu_2$ , can be written as either  $H_0: \mu_1 - \mu_2 = 0$  or  $H_0: \mu_1 = \mu_2$ . A one-sided alternative hypothesis for the difference between population means can be written as either  $H_a: \mu_1 < \mu_2$  (or equivalently  $H_a: \mu_1 - \mu_2 < 0$  or  $H_a: \mu_1 > \mu_2$  (or equivalently  $H_a: \mu_1 - \mu_2 > 0$ )). A two-sided alternative hypothesis for the difference between population means can be written as  $H_a: \mu_1 \neq \mu_2$  (or equivalently  $H_a: \mu_1 - \mu_2 \neq 0$ ).

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**LEARNING OBJECTIVE****4.9.C**

Justify the appropriateness of a hypothesis test for the difference between two population means by verifying conditions.

**[Skill 4.E]****ESSENTIAL KNOWLEDGE****4.9.C.1**

A two-sample  $t$ -test for a difference between population means requires that three conditions be met:

**4.9.C.1.i**

The randomization condition—the data should be collected using two independent random samples or a randomized experiment.

**4.9.C.1.ii**

The 10% condition—when sampling without replacement, the size of each sample should be less than or equal to 10% of the respective population size:  $n_1 \leq 10\%N_1$  and  $n_2 \leq 10\%N_2$ , where  $N_1$  is the size of population 1 and  $N_2$  is the size of population 2. The sample sizes are represented as  $n_1$  and  $n_2$ . (Note: This condition is unnecessary when the data are from a randomized experiment.)

**4.9.C.1.iii**

The sample data condition—both samples should have a sample size greater than or equal to 30 or it is indicated that both population distributions are approximately normal. If either sample size is less than 30, both sample data distributions should be free from strong skewness and outliers.

## SKILLS

## 3.E

Calculate appropriate statistical inference method results.

## 4.F

Interpret results of statistical inference methods.

## 4.G

Justify a claim based on statistical inference method results.

## TOPIC 4.10

# Carrying Out a Test for the Difference Between Two Population Means

## Required Course Content

### LEARNING OBJECTIVE

## 4.10.A

Calculate an appropriate test statistic and  $p$ -value for testing a hypothesis for the difference between two population means.

[Skill 3.E]

## 4.10.B

Interpret the  $p$ -value of a hypothesis test for the difference between two population means.

[Skill 4.F]

### ESSENTIAL KNOWLEDGE

## 4.10.A.1

The test statistic for a two-sample  $t$ -test for the difference between two population means is  $t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$ . The  $t$ -statistic has a

$t$ -distribution when the null hypothesis is true. The  $t$ -statistics with the degrees of freedom can be found using technology. The degrees of freedom fall between  $n_1 + n_2 - 2$  and the smaller of  $n_1 - 1$  and  $n_2 - 1$ .

## 4.10.A.2

The  $p$ -value for a two-sample  $t$ -test for the difference between two population means can be found using the appropriate  $t$ -distribution table or from the appropriate  $t$ -distribution using technology.

## 4.10.B.1

The  $p$ -value is the probability of obtaining a test statistic as extreme or more extreme than the test statistic that was observed (i.e., in the direction of the alternative hypothesis) given that the null hypothesis is true. An interpretation of the  $p$ -value of a hypothesis test for a two-sample test for the difference between two population means should include a statement that the  $p$ -value is computed by assuming that the null hypothesis is true (i.e., by assuming the population means are equal to each other in context).

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**LEARNING OBJECTIVE****4.10.C**

Justify a claim about the populations based on the results of a hypothesis test for the difference between two population means.

**[Skill 4.G]****ESSENTIAL KNOWLEDGE****4.10.C.1**

A formal decision explicitly compares the  $p$ -value to the significance level,  $\alpha$ . If the  $p$ -value  $\leq \alpha$ , then reject the null hypothesis,  $H_0: \mu_1 - \mu_2 = 0$  or  $H_0: \mu_1 = \mu_2$ . If the  $p$ -value  $> \alpha$ , then fail to reject the null hypothesis.

**4.10.C.2**

The results of a hypothesis test for a two-sample  $t$ -test for a difference between two population means can serve as the statistical reasoning to support the answer to an investigative question about the two populations that were sampled.

**4.10.C.3**

A conclusion for the hypothesis test for the difference between two population means is stated in context consistent with, and in terms of, the alternative hypothesis using non-definitive language. The conclusion should contain a reference to the parameters and the populations.

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**AP STATISTICS**

**UNIT 5**

# **Regression Analysis**



**10–20%**  
AP EXAM WEIGHTING



**~9**  
CLASS PERIODS



Remember to go to [AP Classroom](#) to assign students the online **Progress Checks** for this unit.

Whether assigned as homework or completed in class, the **Progress Checks** provide each student with immediate feedback related to this unit's topics and skills.

### **Progress Check 5**

**Multiple-choice: ~19 questions**

**Free-response: 2 questions**

- Question 4: Multi-Focus on Practices 2, 3, and 4
- Question 4: Multi-Focus on Practices 2, 3, and 4

# Regression Analysis

## ESSENTIAL QUESTIONS

- Does the fact that the number of shark attacks increases as ice cream sales increase necessarily mean that ice cream sales cause shark attacks?
- How can we determine the effectiveness of a linear model that uses the number of cricket chirps per minute to predict temperature?



## Developing Understanding

In this unit, students explore relationships in two-variable quantitative data sets. Students describe form, direction, strength, and unusual features for an association between two quantitative variables. They assess correlation and, if appropriate, use a linear model to predict values of the response variable from values of the explanatory variable. Students interpret the least-squares regression line in context, analyze prediction errors (i.e., residuals), and explore departures from a linear pattern.

## Building Statistical Practices

3.A 3.B 4.A 4.B 4.D

In Unit 5, students look at the relationship between variables. The ability to calculate and describe statistical relationships and models is critical for data analysis because students analyze patterns before drawing conclusions about the data. Students should be allowed to use technology to perform calculations so they become more aware of procedural errors. Students will also benefit from having opportunities to practice translating output from technology (i.e., “calculator speak”) into appropriate statistical language. As any statistician would assert, a numerical calculation is only as good as one’s ability to interpret what it means in the real world. Rather than just reporting values from their calculations, students learn to connect their numerical results to the scenario’s context and formulate verbal responses that make the connection clear. Teachers can model good communication and provide high-quality feedback to help students use accurate statistical language to describe a scatterplot, for example, and to avoid common errors in reasoning, such as using the word *line* to explain why a relationship is linear.

## Preparing for the AP Exam

These concepts will be evaluated using individual multiple-choice questions, a multiple-choice question set, and part of a free-response question. Students need ongoing practice with the interpretation of vocabulary and calculated values in context. For example, it is typically insufficient to write generally about the direction of a relationship. If the question is about a linear model for predicting the weight of a wolf based on its length, students are expected to write that a positive relationship means that longer wolves tend to have higher weights. Students can communicate statistical uncertainty by using phrases such as *tend to have* and *on average*, but they need to be careful to use precise language otherwise. For instance, when explaining evidence of a linear relationship, the difference between discussing a rate of change as opposed to merely a change is the difference between right and wrong. For the sake of clarity, teachers can reserve the word *correlation* for discussions about relationships between two quantitative variables.

## UNIT AT A GLANCE

Topic	Skills	Class Periods ~9 CLASS PERIODS
<b>5.1 Graphical Representations Between Two Quantitative Variables</b>	<p><b>3.A</b> Construct tabular and graphical representations of data and distributions.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.B</b> Justify a claim based on statistical calculations and results.</p>	1
<b>5.2 Correlation</b>	<p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p>	2
<b>5.3 Linear Regression Models</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p>	2
<b>5.4 Residuals</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p> <p><b>4.A</b> Describe and compare tabular and graphical representations of data, as well as summary statistics.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p>	2
<b>5.5 Least-Squares Regression</b>	<p><b>3.B</b> Calculate summary statistics, relative positions of points within a distribution, and predicted responses.</p> <p><b>4.D</b> Interpret statistical calculations and results to assess meaning or a claim.</p>	2
 Go to <b>AP Classroom</b> to assign the <b>Progress Check</b> for Unit 5. Review the results in class to identify and address any student misunderstandings.		

## SAMPLE INSTRUCTIONAL ACTIVITIES

The sample activities on this page are optional. They were developed in partnership with teachers from the AP community to share ways that they approach teaching some of the topics in this unit.

Activity	Topics	Sample Activity
1	5.1	<p><b>Sketch and Switch</b></p> <p>Ask students to create a sketch of a scatterplot using a prompt such as a “weak negative, non-linear association.” Then have them switch papers to review each other’s work.</p>
2	5.3	<p><b>Think-Aloud</b></p> <p>Have students toss a handful of coins and record how many land heads up. This is trial 1. Then have them remove the coins that landed heads up. For trial 2, have students toss the remaining coins (i.e., the coins left over after removing those that landed heads up) and record how many land heads up on the second toss. Ask students to think about the trend and make a prediction, discussing their ideas with a partner. Will it be linear? A scatterplot of many trials should show a nonlinear relationship.</p>
3	5.4	<p><b>Reversing Interpretations</b></p> <p>Instead of asking students to interpret a residual, give them the residual and the equation of the least-squares regression line and ask them to make a prediction for a particular observation (e.g., “One wolf in the pack had a length of 1.4 m and a residual of <math>-9.87</math>. What does that <math>-9.87</math> tell us about that particular wolf?”)</p>
4	5.1 5.2 5.3 5.4 5.5	<p><b>Team Challenge</b></p> <p>Set up teams of three or four and task students with solving different multiple-part problems in Unit 5 with different datasets to answer investigative questions. Have teams explain their work to other teams.</p>
5	5.1 5.2 5.3 5.4 5.5	<p><b>Relief Pitcher</b></p> <p>Use this strategy to review multiple concepts at the end of a unit or to spiral review at the end of the course. For instance, when reviewing for the Unit 5 test, students can earn 5 points if they get a question correct when up at the “pitcher’s mound” and 3 points if they do not know the answer but their team in the “bullpen” helps them answer it.</p>

**SKILLS**

**3.A**

Construct tabular and graphical representations of data and distributions.

**4.A**

Describe and compare tabular and graphical representations of data, as well as summary statistics.

**4.B**

Justify a claim based on statistical calculations and results.

**TOPIC 5.1**

# Graphical Representations Between Two Quantitative Variables

## Required Course Content

### LEARNING OBJECTIVE

**5.1.A**

Construct scatterplots depicting the relationship between two quantitative variables.

[Skill 3.A]

**5.1.B**

Describe the characteristics of a scatterplot.

[Skill 4.A]

### ESSENTIAL KNOWLEDGE

**5.1.A.1**

A bivariate quantitative data set consists of observations of ordered pairs from two quantitative variables, collected from the same individuals in a sample or population, and can be used to construct a scatterplot.

**5.1.A.2**

A scatterplot shows the relationship between two quantitative variables for each observation, one corresponding to the value on the  $x$ -axis and one corresponding to the value on the  $y$ -axis. The explanatory variable is placed on the  $x$ -axis and is the variable whose values are used to explain or predict the corresponding values for the response variable, which is placed on the  $y$ -axis.

**5.1.B.1**

A description of the association shown in a scatterplot includes form, direction, strength, and unusual features.

**5.1.B.2**

The form of the association shown in a scatterplot, if any, can be described as linear or non-linear.

**5.1.B.3**

The direction of the association shown in a scatterplot, if any, can be described as positive or negative. A positive association means that as values of the explanatory variable increase, the values of the response variable tend to increase. A negative association means that as values of the explanatory variable increase, the values of the response variable tend to decrease.

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**LEARNING OBJECTIVE**

**5.1.B**

Describe the characteristics of a scatterplot.

[Skill 4.A]

**5.1.C**

Justify a claim using scatterplots depicting the relationship between two quantitative variables.

[Skill 4.B]

**ESSENTIAL KNOWLEDGE**

**5.1.B.4**

The strength of the association shown in a scatterplot is how closely the points follow the general pattern. Strength can be described as strong, moderate, or weak.

**5.1.B.5**

Unusual features of a scatterplot include clusters of individual points or points that don't fit in the general pattern of association between the two variables.

**5.1.C.1**

Scatterplots depicting the relationship between two numeric variables may reveal information that can be used to justify claims about the variable in context.

## SKILLS

## 4.D

Interpret statistical calculations and results to assess meaning or a claim.

## TOPIC 5.2

# Correlation

### Required Course Content

#### LEARNING OBJECTIVE

**5.2.A**

Interpret the correlation for a linear relationship.

[Skill 4.D]

#### ESSENTIAL KNOWLEDGE

**5.2.A.1**

The correlation coefficient,  $r$ , summarizes the strength and direction of the linear association between two quantitative variables. The correlation coefficient  $r$  is unit-free and always between  $-1$  and  $1$ , inclusive. A negative correlation coefficient value indicates a negative association, and a positive correlation coefficient value indicates a positive association.

**5.2.A.2**

The strength of the linear association is determined by how close the correlation coefficient is to  $-1$  or  $1$ . A value of  $r = 0$  indicates that there is no linear association. A value of  $r = -1$  or  $r = 1$  indicates that there is a perfect linear association.

**5.2.A.3**

A correlation coefficient close to  $-1$  or  $1$  does not necessarily mean that a linear model is appropriate.

**5.2.A.4**

A perceived or real relationship between two variables does not mean that changes in one variable cause changes in the other. That is, correlation does not necessarily imply causation.

## TOPIC 5.3

# Linear Regression Models

## SKILLS

## 3.B

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

## Required Course Content

### LEARNING OBJECTIVE

## 5.3.A

Calculate a predicted response value using a linear regression model.

[Skill 3.B]

### ESSENTIAL KNOWLEDGE

## 5.3.A.1

If the form of the relationship between  $x$  and  $y$  appears linear, we can approximate the relationship between  $x$  and  $y$  using a linear regression model, which is a linear equation that uses an explanatory variable,  $x$ , to predict the response variable,  $y$ .

## 5.3.A.2

In a linear regression model, the predicted response value, denoted by  $\hat{y}$ , is calculated as  $\hat{y} = a + bx$ , where  $a$  is the  $y$ -intercept,  $b$  is the slope of the regression line, and  $x$  is the explanatory variable.

## 5.3.A.3

Extrapolation is predicting a response value using a value for the explanatory variable that is beyond the interval of  $x$ -values used to determine the regression line. The predicted value is less reliable the further the estimate is extrapolated.

## 5.3.A.4

Interpolation is predicting a response value using a value for the explanatory variable that is within the interval of  $x$ -values used to determine the regression line.

## SKILLS

## 3.B

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

## 4.A

Describe and compare tabular and graphical representations of data, as well as summary statistics.

## 4.D

Interpret statistical calculations and results to assess meaning or a claim.

## TOPIC 5.4

# Residuals

### Required Course Content

#### LEARNING OBJECTIVE

## 5.4.A

Calculate the differences between the observed and predicted values.

[Skill 3.B]

## 5.4.B

Interpret the differences between the observed and predicted values.

[Skill 4.D]

## 5.4.C

Describe the form of association of bivariate data using residual plots.

[Skill 4.A]

#### ESSENTIAL KNOWLEDGE

## 5.4.A.1

A residual is the difference between the observed response value and the predicted response value for the given value of the explanatory variable:  $\text{residual} = y - \hat{y}$  or ( $\text{residual} = \text{observed } y - \text{predicted } y$ ).

## 5.4.B.1

If the residual is positive, the model underpredicts (underestimates) the value of the response variable. If the residual is negative, the model overpredicts (overestimates) the value of the response variable.

## 5.4.C.1

A residual plot is a scatterplot of the residuals versus the predicted response values (or the explanatory variable values).

## 5.4.C.2

Residual plots can be used to investigate the appropriateness of the linear regression model for the observed data.

## 5.4.C.3

The linear regression model should only be fit to the data if the data exhibit a linear trend. Apparent randomness in a residual plot for a linear regression model is confirmation of a linear form in the association between the two variables and indicates that the simple linear regression model is an appropriate model for the data.

## 5.4.C.4

Curvature in the residual plot for a linear regression model suggests that the linear model is not the most appropriate model for the data.

## TOPIC 5.5

Least-Squares  
Regression

## SKILLS

## 3.B

Calculate summary statistics, relative positions of points within a distribution, and predicted responses.

## 4.D

Interpret statistical calculations and results to assess meaning or a claim.

## Required Course Content

## LEARNING OBJECTIVE

## 5.5.A

Calculate the coefficients for the least-squares regression line model.

[Skill 3.B]

## 5.5.B

Interpret coefficients for the least-squares regression line model.

[Skill 4.D]

## ESSENTIAL KNOWLEDGE

## 5.5.A.1

The simple linear regression model is fit to the data by minimizing the sum of the squares of the residuals. Because of this, the resulting equation is often called the least-squares regression line (*LSRL*) and is calculated using technology. This regression line will pass through the point  $(\bar{x}, \bar{y})$ .

## 5.5.A.2

The slope of the regression line,  $b$ , is calculated using technology.

## 5.5.A.3

The  $y$ -intercept of the regression line,  $a$ , is calculated using technology.

## 5.5.A.4

In simple linear regression, the correlation coefficient,  $r$ , is calculated using technology.

## 5.5.A.5

In simple linear regression, the square of the correlation coefficient,  $r^2$ , is called the coefficient of determination. The value of  $r^2$  is the proportion of variation in the response variable that is explained by the linear relationship with the explanatory variable.

## 5.5.B.1

The coefficients of the least-squares regression line model (line of best fit) are the slope,  $b$ , and the  $y$ -intercept,  $a$ , because they are based on a sample of values.

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## LEARNING OBJECTIVE

**5.5.B**

Interpret coefficients for the least-squares regression line model.

[Skill 4.D]

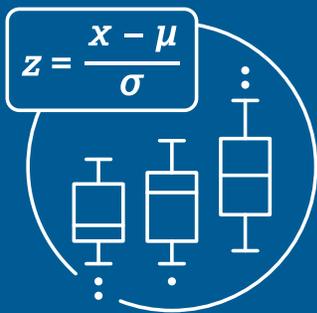
## ESSENTIAL KNOWLEDGE

**5.5.B.2**

The slope of the least-squares regression line can be interpreted as the predicted increase or decrease in the response variable for a one-unit increase in the explanatory variable, and it should be interpreted in context.

**5.5.B.3**

The  $y$ -intercept in the least-squares regression line is the predicted value of the response variable when the explanatory variable is equal to 0, and it should be interpreted in context. Sometimes, the  $y$ -intercept of the line does not have a reasonable interpretation in context because  $x = 0$  might be beyond the interval of  $x$ -values used to determine the regression line (extrapolation). At other times, the  $y$ -intercept of the line does not have a logical interpretation in context because it might be a negative value for a response variable that has no negative values, such as height.



AP STATISTICS

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# Exam Information

# Exam Overview

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The AP Statistics Exam assesses student understanding of the statistical practices and learning objectives outlined in the course framework. The exam is 3 hours long and includes 42 multiple-choice questions and four 10-point free-response questions, each weighted equally and scored on an analytic scale. A graphing calculator with statistical capabilities is expected for both sections of the exam. Formulas and tables are provided for both sections of the exam (see the [Appendix](#)). The details of the exam, including exam weighting and timing, can be found below:

Section	Question Type	Number of Questions	Exam Weighting	Timing
I	Multiple-choice questions	42	50%	90 minutes
II	Free-response questions	4	50%	90 minutes
	Question 1: Multi-Focus on Practices 1 and 2	1	12.5%	
	Question 2: Multi-Focus on Practices 3 and 4	1	12.5%	
	Question 3: Inference (Hypothesis Test or Confidence Interval)	1	12.5%	
	Question 4: Multi-Focus on Practices 2, 3, and 4	1	12.5%	

Accessible technology that has the capabilities expected for AP Statistics is available for students who are blind or visually impaired. This technology should be used during the course, and an accommodation request to use this technology on the AP Exam must be made through the College Board's [Services for Students with Disabilities \(SSD\)](#).

# How Student Learning Is Assessed on the AP Exam

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## Section I: Multiple-Choice

The first section of the AP Statistics Exam includes 42 multiple-choice questions comprised of discrete and set-based questions. Students are expected to use a graphing calculator with statistical capabilities. Questions may focus on scenarios related to a variety of work applications and/or real-life contexts. All five units of the course are assessed in the multiple-choice section of the AP Statistics Exam, with the following exam weighting.

Units of Instruction	Approximate Exam Weighting for Multiple-Choice Section
Unit 1: Exploring One-Variable Data and Collecting Data	20–30%
Unit 2: Probability, Random Variables, and Probability Distributions	15–25%
Unit 3: Inference for Categorical Data: Proportions	15–25%
Unit 4: Inference for Quantitative Data: Means	10–20%
Unit 5: Regression Analysis	10–20%

All four statistical practices are assessed in the multiple-choice section of the AP Statistics Exam, with the following exam weighting.

Statistical Practices	Approximate Exam Weighting for Multiple-Choice Section
Practice 1: Formulate Questions	5–10%
Practice 2: Collect Data	20–30%
Practice 3: Analyze Data	25–35%
Practice 4: Interpret Results	25–35%

**Practice 1: Formulate Questions.** These questions assess students' ability to determine an investigative question for a statistical study. Students must determine a valid investigative question that requires a statistical investigation.

**Practice 2: Collect Data.** These questions assess students' ability to identify and justify methods for collecting data and conducting statistical inference. Students must identify information that helps them answer a question or solve a problem and justify an appropriate method for ethically gathering and representing data. Students also identify appropriate statistical inference methods, null and alternative hypotheses, types of errors, and relationships among components in statistical inference methods.

**Practice 3: Analyze Data.** These questions assess students' ability to construct representations of data and calculate numerical statistical outputs. Students must construct tabular and graphical representations of data and distributions. Students calculate summary statistics, relative positions of points within a distribution, and predicted responses. Additionally, students calculate and estimate expected counts, percentages, probabilities, and intervals. For probability distributions, students calculate means, standard deviations, and parameters. This practice also requires that students calculate appropriate statistical inference method results.

**Practice 4: Interpret Results.** These questions assess students' ability to interpret results and justify conclusions and methods. Students must describe and compare tabular and graphical representations of data. Students should also know how to justify a claim based on statistical calculations and results. This practice also focuses on describing distributions, comparing relative positions of points within a distribution, and interpreting statistical calculations and results to assess a meaning or claim. For inference procedures, students justify the use of a chosen statistical inference method by verifying conditions, interpreting the results of statistical inference methods, and justifying a claim based on statistical inference method results.

## Section II: Free-Response

The second section of the AP Statistics Exam includes four 10-point question types, which are listed below in the order they appear on the exam. Students are expected to use a graphing calculator that has statistical capabilities.

**Question 1: Multi-Focus on Practices 1 and 2** is a multi-part question that primarily assesses Practices 1 and 2.

**Question 2: Multi-Focus on Practices 3 and 4** is a multi-part question that primarily assesses Practices 3 and 4.

**Question 3: Inference** is a question with a focus on inference, assessing the inference skills associated with Practices 2, 3, and 4.

**Question 4: Multi-Focus on Practices 2, 3, and 4** is a multi-part question with a focus on multiple course content areas, assessing Practices 2, 3, and 4.

# Task Verbs Used in Free-Response Questions

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The following **task verbs** are commonly used in the free-response questions:

**Calculate:** Perform mathematical steps to arrive at a final answer (e.g., statistical calculations).

**Compare:** Provide a description or explanation of similarities and/or differences. Numerically, this task verb requires a quantitative comparison of two or more values. In terms of distributions or graphical representations, this task verb requires identifying aspects of two or more distributions or graphical representations that are similar and that are different.

**Complete:** Identify parts, justify conditions, and calculate the results for the appropriate statistical inference methods.

**Construct:** Represent data in graphical or tabular form.

**Describe:** Provide the relevant characteristics of representations.

**Determine:** Apply an appropriate method, investigative question, or a claim based on calculations and/or findings.

**Estimate:** Use models or representations to find approximations.

**Explain:** Provide information about how or why a relationship, process, position, situation, or outcome occurs, using evidence and/or reasoning to support or qualify a claim.

**Identify/Classify:** Indicate or provide information about a specified topic in words without elaboration or explanation.

**Interpret:** Describe the connection between a mathematical expression, representation, or solution and its meaning within the realistic context of a problem.

**Justify:** Provide evidence to support, qualify, or defend a claim, and/or provide statistical reasoning to explain how that evidence supports or qualifies the claim.

# Sample Exam Questions

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The sample exam questions that follow illustrate the relationship between the course framework and the AP Statistics Exam and serve as examples of the types of questions that appear on the exam. These sample questions do not represent the full range and distribution of items on an official AP Statistics Exam. After the sample questions is a table that shows which skills, units, learning objectives, and essential knowledge statements each question assesses. The table also provides the answers to the multiple-choice questions.

## Section I: Multiple-Choice

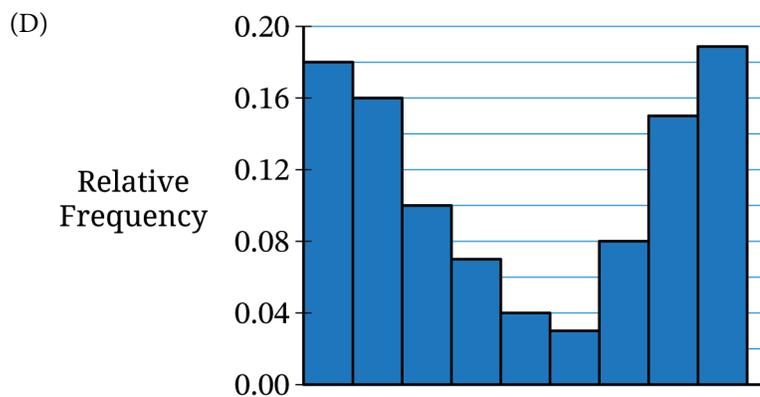
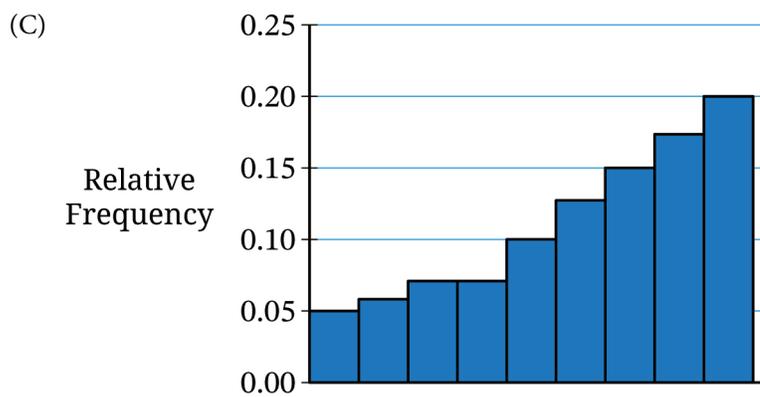
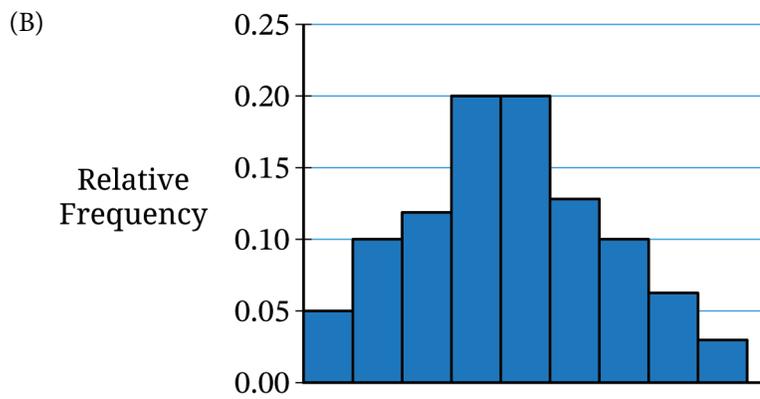
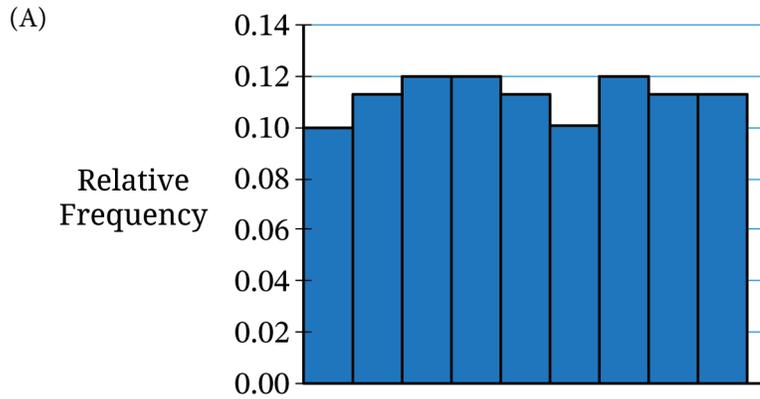
The following are examples of the kinds of multiple-choice questions found on the exam.

1. A social media researcher wants to study the average daily screen time (time spent using electronic devices such as mobile phones, computers, or game consoles) of teenagers, 13 to 17 years of age, in a large country. Which of the following describes why a statistical study is necessary for this research?
  - (A) A statistical study is always required for research.
  - (B) A statistical study would avoid collecting confidential data from teenagers.
  - (C) A statistical study would ensure that every teenager in the large country participates in the study.
  - (D) A statistical study would be used because surveying every member of the teenager population would be too difficult.
2. A library provides two methods for borrowing videos. Library patrons (people who use the library) can borrow videos either in-person or by using an online system. One of the librarians randomly selects patrons who borrow videos to gather data on how the patrons feel about these two methods.

Which of the following describes the sample and the population for the librarian's study?

  - (A) The sample consists of the patrons selected for the study who borrow videos; the population is all patrons of the library.
  - (B) The sample consists of the patrons selected for the study who borrow videos; the population is all patrons of the library who borrow videos.
  - (C) The sample consists of the patrons selected for the study who borrow videos using the in-person method; the population is all patrons of the library who borrow videos.
  - (D) The sample consists of the patrons selected for the study who borrow videos using the online system; the population is all patrons of the library who borrow videos.

3. Which of the following histograms has a shape that is approximately uniform?



4. A doctor wanted to publish a research study about newborn babies in a certain city. She randomly selected newborn babies from the city and recorded their weights, in kilograms, using the same scale. She later discovered that the scale she used was incorrectly adding 0.25 kilogram to the weight of each newborn baby. To correct the error, she adjusted each of the weights in the random sample by subtracting 0.25 kilogram.

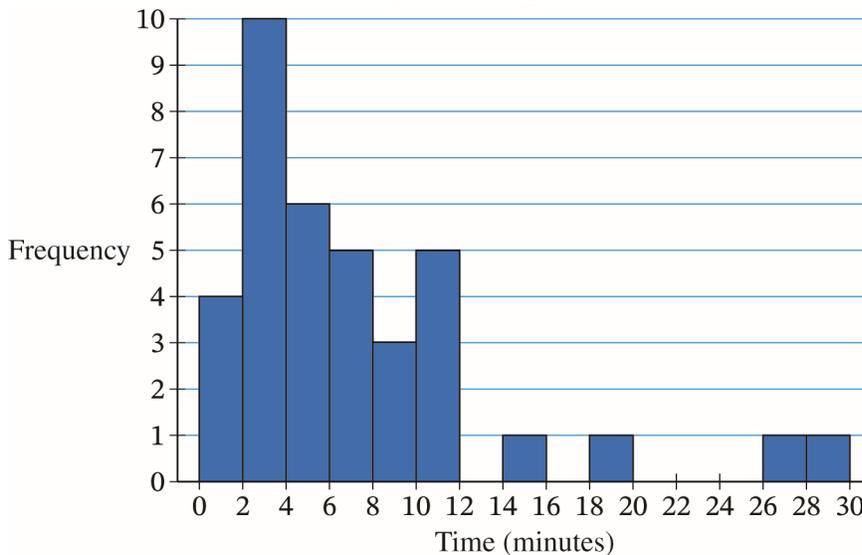
Which of the following statistics for the weights, in kilograms, of the newborn babies has the same value before and after the doctor's adjustment?

- (A) Median  
 (B) Mean  
 (C) Third quartile  
 (D) Interquartile range
5. During a 90-minute lecture, a teaching assistant recorded the amount of time, in minutes, that each student spent looking at their cell phone. A table displaying some summary statistics and a histogram displaying the results are shown.

**Summary Statistics**

<i>n</i>	Mean	Standard Deviation	Q1	Q3
37	6.865	6.447	2	9.5

**Time Spent Looking at Cell Phone**



Based on the information given and the  $1.5 \times \text{IQR}$  rule, which of the following conclusions can be made about the median and the number of outliers?

- (A) The median is greater than 6.865, and the distribution has exactly two outliers.  
 (B) The median is greater than 6.865, and the distribution has exactly four outliers.  
 (C) The median is less than 6.865, and the distribution has exactly two outliers.  
 (D) The median is less than 6.865, and the distribution has exactly four outliers.

6. An athletic trainer believes that a new natural treatment will decrease the mean healing time, in days, compared to the current treatment for athletes with knee injuries. The athletic trainer would like to create a study to test his belief. The mean healing time for athletes receiving the current treatment is 21 days. An investigative question will be formulated before the study is created.

Which of the following would be the appropriate components of the athletic trainer's investigative question?

- (A) The parameter is the mean healing time for knee injuries when using the new natural treatment, and the direction of the alternative hypothesis is not equal to 21.
  - (B) The parameter is the mean healing time for knee injuries when using the new natural treatment, and the direction of the alternative hypothesis is less than 21.
  - (C) The parameter is the healing time for knee injuries when using the new natural treatment, and the direction of the alternative hypothesis is not equal to 21.
  - (D) The parameter is the healing time for knee injuries when using the new natural treatment, and the direction of the alternative hypothesis is less than 21.
7. A company is hired to identify the issues that all residents in a large district are concerned about. The company divides the district into twenty regions and believes that the regions are similar to one another in their composition of residents. The company then randomly selects two of the regions and surveys all residents in both regions.

Which of the following is the sampling method that will be used by the company?

- (A) Simple random sampling
- (B) Stratified random sampling
- (C) Systematic random sampling
- (D) Cluster random sampling

8. A company blood drive collected 42 pints of donated blood. The table shows the classification of each of the pints based on blood type (O, A, B, or AB) and Rh factor (positive or negative).

**Blood Type Classification**

	Type O	Type A	Type B	Type AB	Total
Negative	3	4	1	0	8
Positive	16	13	4	1	34
Total	19	17	5	1	42

Based on the table, of all pints of blood classified as positive, approximately what proportion is blood type A?

- (A) 0.31
- (B) 0.38
- (C) 0.50
- (D) 0.76

**Questions 9–11 refer to the following.**

A teacher has noticed that many students are left-handed, and they often struggle when using right-handed scissors. A research article indicated that the percentage of people who are left-handed is approximately 10% worldwide.

9. If random samples of 20 students are taken from classes around the world, which of the following is the correct standard deviation for the distribution of the number of students who are left-handed?
- (A) 0.005
  - (B) 0.067
  - (C) 1.342
  - (D) 1.800
10. What is the probability that from one sample of 20 randomly selected students at least 4 of the students will be left-handed?
- (A) 0.0432
  - (B) 0.0898
  - (C) 0.1330
  - (D) 0.9568
11. For random samples of 20 students, the mean is 2. Which of the following is a correct interpretation for the mean?
- (A) For all possible classes of size 20, the average number of left-handed students is 2 students.
  - (B) For all possible classes in the teacher's school, the average number of left-handed students is 2 students.
  - (C) Every class of 20 students will have 2 students who are left-handed.
  - (D) Every class of students will have 2 students who are left-handed.

12. For a certain population of penguins, the distribution of the weight of a penguin is approximately normal with a mean of 15.1 kilograms (kg) and a standard deviation of 2.2 kg. What is the probability that a randomly selected penguin from this population weighs between 13.0 kg and 16.5 kg?
- (A) 0.17  
(B) 0.26  
(C) 0.43  
(D) 0.57
13. Mark constructed a 90% confidence interval for the mean commute time for commuters in his town. Which of the following statements will be correct if he reconstructs the confidence interval with the same sample data at a 95% level of confidence?
- (A) The width of the confidence interval would increase.  
(B) The width of the confidence interval would decrease.  
(C) The margin of error would remain the same.  
(D) The standard error would decrease.
14. A principal claims that more than 50% of the students in a certain school exercise at least 3.5 hours per week. To investigate the claim, the principal selected a random sample of 40 students in the school and found that 60% of the students in the sample exercise at least 3.5 hours per week. Let  $p$  represent the proportion of all students in the school who exercise at least 3.5 hours per week. Which of the following hypotheses are appropriate to test the principal's claim?
- (A)  $H_0 : p = 0.5$  and  $H_a : p > 0.5$   
(B)  $H_0 : \hat{p} = 0.5$  and  $H_a : \hat{p} \neq 0.5$   
(C)  $H_0 : p = 0.6$  and  $H_a : p > 0.6$   
(D)  $H_0 : \hat{p} = 0.6$  and  $H_a : \hat{p} > 0.6$
15. A one-sample  $z$ -test for a proportion at the significance level of  $\alpha = 0.10$  will be conducted for the following set of hypotheses.

$$H_0 : p = 0.50$$

$$H_a : p < 0.50$$

Which of the following statements represents a Type II error for the hypotheses?

- (A) Rejecting the null hypothesis when the population proportion is 0.50  
(B) Rejecting the null hypothesis when the population proportion is less than 0.50  
(C) Failing to reject the null hypothesis when the population proportion is 0.50  
(D) Failing to reject the null hypothesis when the population proportion is less than 0.50

16. A survey was conducted to investigate whether the proportion of all teenagers, 13–17 years of age, who sleep with a blanket in their bed is different than the proportion of all adults who sleep with a blanket in their bed. A random sample of 250 teenagers and a random sample of 325 adults were selected. Of the 250 teenagers selected, 70 indicated they sleep with a blanket. Of the 325 adults selected, 39 indicated they sleep with a blanket. A two-sample  $z$ -interval for the difference in the proportion of all teenagers who sleep with a blanket and the proportion of all adults who sleep with a blanket was created from the survey results.

Which of the following describes how the normality condition is being met for the two-sample  $z$ -interval for a difference between two population proportions?

- (A) A random sample of teenagers and a random sample of adults were used to collect the data.
  - (B) The sample size of teenagers and the sample size of adults were less than 10% of their respective populations.
  - (C) The sample size of teenagers and the sample size of adults were greater than 30.
  - (D) The observed number of successes and failures for each sample were greater than 10.
17. In 2022, 17% of a random sample of 200 adults in a certain region indicated they worked from home that year. In 2024, 25% of a random sample of 600 adults in the same region indicated they worked from home that year. A two-sample test was conducted to determine if the population proportion of adults in the region who worked from home was greater in 2024 than in 2022. Assume all conditions for inference were met.

Which of the following is the test statistic and  $p$ -value of the test conducted?

- (A)  $z = 2.33$  and  $p$ -value = 0.0099
  - (B)  $z = 2.33$  and  $p$ -value = 0.9901
  - (C)  $t = 2.33$  and  $p$ -value = 0.0099
  - (D)  $t = 2.33$  and  $p$ -value = 0.9901
18. A physician assistant who specializes in sports medicine conducted a study to investigate whether there is an association between swimming experience and the occurrence of a certain sport injury for swimmers. Data were collected on a random sample of 51 swimmers. Each swimmer from the sample was categorized by swimming experience (low, medium, high) and whether or not the swimmer experienced the sport injury.

Which of the following is the appropriate test for the physician assistant's study?

- (A) A one-sample  $z$ -test for a population proportion
- (B) A two-sample  $z$ -test for the difference between two population proportions
- (C) A chi-square test for independence
- (D) A chi-square test for homogeneity

19. The distribution of ages for older adults at a large recreation center is unimodal and roughly symmetric with a mean of 65 years and a standard deviation of 10.6 years. Consider a simulation with 200 trials in which, for each trial, the ages from a random sample of 30 older adults are selected and the mean age is calculated.

Which of the following statements is a correct description for the distribution of the 200 sample mean ages?

- (A) Approximately normal with mean 65 years and standard deviation 10.6 years
  - (B) Approximately normal with mean 65 years and standard deviation 1.94 years
  - (C) Approximately uniform with mean 65 years and standard deviation 0.75 years
  - (D) Approximately uniform with mean 65 years and standard deviation 1.94 years
20. An agricultural engineer selected a random sample of 30 farms in a country to construct a 95% confidence interval for the mean size, in acres, of farms in the country. The resulting interval was (367, 558).

Which of the following is a correct interpretation of the 95% confidence level?

- (A) Approximately 95% of all farm sizes in the country are between 367 acres and 558 acres.
- (B) Approximately 95% of all random samples of size 30 from the population will produce a sample mean farm size between 367 acres and 558 acres.
- (C) Approximately 95% of all random samples of size 30 from the population will produce intervals that contain the true mean farm size, in acres, from the country.
- (D) Approximately 95% of all random samples of size 30 from the population will produce intervals that contain the sample mean farm size, in acres, from the country.

21. The recommended daily amount of a vitamin for women is 18 milligrams (mg) per day. A researcher studying the nutritional needs of women living in a certain region suspected that the women were getting less than the recommended daily amount on average. The researcher took a random sample of women from the region and measured their daily intake (amount taken) of the vitamin. The following hypotheses were tested at the significance level of  $\alpha = 0.05$  for the population mean,  $\mu$ , of the daily intake of the vitamin for women in the region.

$$H_0 : \mu = 18$$

$$H_a : \mu < 18$$

All conditions for inference were met, and the resulting  $p$ -value was 0.031.

Which of the following is an appropriate conclusion for the test?

- (A) The  $p$ -value of 0.031 is less than  $\alpha = 0.05$ , and the null hypothesis should be rejected. There is convincing statistical evidence that the mean daily intake of the vitamin for women in the region is less than the recommended 18 mg.
- (B) The  $p$ -value of 0.031 is less than  $\alpha = 0.05$ , and the null hypothesis should be rejected. There is not convincing statistical evidence that the mean daily intake of the vitamin for women in the region is less than the recommended 18 mg.
- (C) The  $p$ -value of 0.031 is less than  $\alpha = 0.05$ , and the null hypothesis should be rejected. There is convincing statistical evidence that proves that the mean daily intake of the vitamin for women in the region is less than the recommended 18 mg.
- (D) The  $p$ -value of 0.031 is less than  $\alpha = 0.05$ , and the null hypothesis should not be rejected. There is not convincing statistical evidence that the mean daily intake of the vitamin for women in the region is the recommended 18 mg.

**Questions 22–24 refer to the following.**

Emmalyn runs track-and-field at a local running club. To complete a project in her statistics class, she collects data from her teammates, measuring their 60-meter-dash time, in seconds, and long-jump distance, in inches. Using these data, she created a linear regression model using dash time as the  $x$ -variable and long-jump distance as the  $y$ -variable. The model she created is  $\hat{y} = 313.875 - 30.805x$  and has  $r = -0.591$  and  $r^2 = 0.349$ .

22. Which of the following is a correct interpretation of the correlation coefficient?
- (A) There is no linear relationship between dash time and long-jump distance.
  - (B) The linear relationship between dash time and long-jump distance is moderate and negative.
  - (C) There is a perfect linear relationship between dash time and long-jump distance.
  - (D) The linear relationship between dash time and long-jump distance is moderate and positive.

23. Which of the following is a correct interpretation for the coefficient of determination?
- (A) Approximately 35% of the variation in dash time can be explained by long-jump distance.
  - (B) Approximately 59% of the variation in dash time can be explained by long-jump distance.
  - (C) Approximately 35% of the variation in long-jump distance can be explained by the linear relationship with dash time.
  - (D) Approximately 59% of the variation in long-jump distance can be explained by the linear relationship with dash time.
24. A teammate with a dash time of 7 seconds had a residual of  $-6.8$  inches. Based on the model, what is the observed long-jump distance of this teammate, in inches?
- (A) 91.44
  - (B) 98.24
  - (C) 105.04
  - (D) 313.88

## Section II: Free-Response

The following are examples of the kinds of free-response questions found on the exam.

### QUESTION 1

A country's transportation department wants to conduct a survey to collect data from adults, ages 18 and older, in that country. The survey will be focused on commuting to work, including commute time in minutes, method of transportation used, and other information.

Country A is divided into many regions, and each region will be asked to send out a survey and collect the data. The transportation department will allow each region to use any method it preferred to survey the adults.

Three different regions will use the following sampling methods:

Region X: Each adult who lives in the region will receive a survey and will be required to complete the survey by mail. Adults who do not complete the survey will be sent reminders to do so until they have completed the survey.

Region Y: The region is divided into six areas that are similar to each other. Two of the six areas will be randomly selected, and each adult who lives in those two areas will receive a survey and be asked to complete the survey by mail. The table gives the results of the data collection for the two areas in Region Y for the question "What is your average daily commute time to work in minutes?"

Region Z: Each adult who lives in the region and has responded to a previous official survey will receive a survey and be asked to complete it by mail.

Summary Statistics

Area	$n$	$\bar{x}$	Min	Q1	Median (Q2)	Q3	Max
One	2,887	55	7	35	53	74	101
Five	4,502	42	5	24	32	68	83

Use the given information to respond to parts A, B, C, D, and E. Label any subparts (e.g., i and ii) that may be present.

A.

- Identify** the sampling method for Region X.
- Identify** the sampling method for Region Y.

B.

- Explain** why the results of the sampling method for Region Y can be generalized to the population of the region.
- Tom, a member of the planning committee for Region Z, proposed Region Z's sampling method. Tom claims the sampling method for Region Z may lead to a higher survey response rate than the sampling methods used for the other two regions and could better represent the entire population in Region Z. **Explain** why Tom's claim is not correct.

- C. Manuel, an insurance agent in Region Y, wants to compare the average commute times to work for the populations in each of the two areas of Region Y. His first step is to construct a graphical representation for each area using the summary statistics in the table.
- Identify** a graphical representation that is appropriate to use to display the summary statistics.
  - Explain** why the graphical representation from part C (i) is appropriate to use.
- D. Use comparisons of the summary statistics in the table to **describe** the most likely shape for the distribution of commute time in Area Five.
- E. Manuel wants to perform a study about the average daily commute time to work for Area Three and Area Six in Region Y. He believes there is a difference in the commute times for the two areas. **Determine** a valid investigative question for Manuel's study. Ensure all three components of a valid investigative question are included in your response.

## QUESTION 2

Andrea, a cybersecurity specialist at a bank, was concerned about the security of the bank's website. She was asked to compare two encryption (a process where information is scrambled and cannot be easily read) methods that could be used to protect the website's banking information. Her department was concerned that the older encryption version, DES, may be weaker than the newer encryption version, AES. One measure of how well an encryption method works is session duration (how long the information is accessed), in seconds. A very long session duration may indicate that there has been unauthorized access to the data, which means that the encryption method needs to be more secure. Her department wanted her to study the two encryption methods.

Therefore, Andrea conducted a study of the two types of encryption methods. Andrea randomly collected this information from bank customers who used the website over the previous six months. For each session of the sampled customers, Andrea collected the session duration, in seconds, and the type of encryption method used. The table displays the summary statistics for her study.

**Summary Statistics**

Encryption Method	$n$	$\bar{x}$	$s$	Min	Q1	Median (Q2)	Q3	Max
DES	2,865	782	768	1	226	543	1,107	6,950
AES	4,706	794	803	1	233	794	1,091	7,190

Use the given information to respond to parts A, B, C, D, and E. Label any subparts (e.g., i and ii) that may be present.

- A.
- Identify** the type of study Andrea conducted.
  - Identify** the categorical variable in Andrea's study.

- B. Use the table to respond to the following. Show the work that leads to your answers.
- Determine** whether there are outliers using the  $1.5 \times \text{IQR}$  rule for the DES encryption method.
  - Determine** whether there are outliers using the  $1.5 \times \text{IQR}$  rule for the AES encryption method.
- C. Use the table to respond to the following.
- Compare** the distance from Q1 to the median and the distance from the median to Q3 for the DES encryption method.
  - Compare** the distance from Q1 to the median and the distance from the median to Q3 for the AES encryption method.
  - Explain** how these comparisons can be used to select which measure of variability—standard deviation, range, or IQR—would be the best representation for both of the encryption methods.
- D. Andrea decides to compare the medians of the two encryption methods to determine which encryption method is best. Use the table and your responses to part B and part C to **determine** whether Andrea is using the correct measure of center to compare the distributions. **Justify** your answer using both of the encryption methods.
- E. **Determine** the population for a valid investigative question for Andrea’s study.

### QUESTION 3

Neil, a marketing director, wanted to compare the effectiveness of two different video advertisements designed to persuade people to buy a new laptop. He showed the two advertisements, version A and version B, to different groups of people. The responses from the different groups of people were compared to determine which version performs better. Each advertisement contains a unique code that people can text using their cell phones to get more information about the laptop. Neil will use the advertisement that has a higher percentage of people who text that version’s code. If there is no difference in the percentages for the two advertisements, he will use both.

Neil gathered a group of 500 volunteers who were interested in purchasing a laptop. The volunteers were randomly assigned to groups. One group watched version A of the advertisement for the new laptop, and the other group watched version B of the advertisement for the new laptop. For version A, 83 of the 250 volunteers texted the code to get more information. For version B, 54 of the 250 volunteers texted the code to get more information.

At the 0.05 level of significance, complete the appropriate inference procedure to determine if there is convincing statistical evidence of a difference in the proportion of adults similar to those in the study who will watch version A and text the code than the proportion of adults similar to those in the study who will watch version B and text the code.

Use the given information to respond to parts A, B, C, and D.

- Identify** and **classify** the variable(s) in the study.
- Identify** the appropriate inference procedure.

- C. **Complete** the inference procedure, including **calculating** the appropriate statistics.
- D. **Justify** a conclusion in context.

#### QUESTION 4

According to a 2024 national survey of 400 randomly selected offices in a certain country, the average number of water bottle refilling stations was 2.6 with a standard deviation of 1.6 stations. Erika is a marketer at a water bottle company, and she wants to investigate whether the average number of refilling stations per office has changed from 2024 to 2025. In 2025, she selected a random sample of 350 offices in the country and asked how many refilling stations were in each office. The results of the 2025 sample are summarized in the table.

**Number of Water Bottle Refilling Stations and Proportions of Offices in 2025**

Number of Refilling Stations	0	1	2	3	4	5	6
Proportion of Offices	0.04	0.13	0.21	0.35	0.19	0.05	0.03

Erika also found the standard deviation for the sample to be 1.31 refilling stations.

Use the given information to respond to parts A, B, C, D, and E. Label any subparts (e.g., i and ii) that may be present.

- A. Use the table to respond to the following. Show all work for calculations.
  - i. An office from the sample will be selected at random. **Calculate** the probability that an office had 3 or more refilling stations in 2025.
  - ii. **Calculate** the average number of refilling stations per office for the sample of offices in 2025.
- B. Erika believes that the average number of refilling stations per office for the sample of offices in 2025 has changed compared to the sample of offices from 2024. **Determine** whether Erika's belief is supported by the data. Use the results from part A (ii) to **justify** your answer without conducting an inference procedure.
- C. Erika wants to conduct an inference procedure to estimate the difference in the true mean number of refilling stations per office between 2024 and 2025. **Identify** the appropriate inference procedure Erika should use.
- D. Assume the conditions for the inference procedure identified in part C were satisfied and that Erika used a 90% confidence level.
  - i. **Calculate** the results from the appropriate inference procedure.
  - ii. **Interpret** a conclusion from part D (i) in context.
- E. Based on the results from Erika's inference procedure, **determine** whether there is sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025. **Explain** your reasoning.

# Answer Key and Question Alignment to Course Framework

Multiple-Choice Question	Answer	Skill	Unit	Learning Objective	Essential Knowledge
1	D	2.A	1	1.1.A	1.1.A.2
2	B	2.A	1	1.1.A	1.1.A.4, 1.1.A.5
3	A	4.A	1	1.6.A	1.6.A.3
4	D	3.B	1	1.7.B, 1.7.C	1.7.B.3, 1.7.C.1
5	C	3.B, 4.A	1	1.7.D, 1.8.B	1.7.D.1, 1.8.B.1
6	B	1.A	1	1.10.A	1.10.A.2
7	D	2.A	1	1.11.A	1.11.A.5
8	B	3.B	2	2.2.A	2.2.A.3
9	C	3.D	2	2.10.B	2.10.B.1
10	C	3.C	2	2.10.E	2.10.E.1
11	A	4.D	2	2.10.C	2.10.C.1
12	D	3.C	2	2.11.E	2.11.E.2
13	A	2.D	4	4.3.C	4.3.C.1
14	A	2.E	3	3.5.B	3.5.B.3
15	D	2.D	3	3.8.A	3.8.A.2
16	D	4.E	3	3.10.B	3.10.B.1
17	A	3.E	3	3.13.A	3.13.A.1, 3.13.A.2
18	C	2.C	3	3.14.B	3.14.B.3
19	B	3.D	4	4.1.A	4.1.A.1
20	C	4.F	4	4.3.A	4.3.A.2
21	A	4.G	4	4.5.C	4.5.C.1, 4.5.C.3
22	B	4.D	5	5.2.A	5.2.A.1, 5.2.A.2
23	C	3.B	5	5.5.A	5.5.A.4
24	A	3.B	5	5.4.A	5.4.A.1

Free-Response Question	Skills	Units	Learning Objectives	Essential Knowledge
1	1.A, 2.A, 2.B, 3.A, 4.A	1	1.6.A, 1.8.A, 1.10.A, 1.10.B, 1.10.E, 1.11.A	1.6.A.1, 1.8.A.2, 1.10.A.1, 1.10.A.2, 1.10.A.3, 1.10.B.1, 1.10.E.2, 1.10.E.4, 1.11.A.5
2	1.A, 2.A, 3.B, 4.A, 4.B	1	1.2.B, 1.6.A, 1.7.D, 1.7.F, 1.10.A, 1.10.D	1.2.B.1, 1.6.A.1, 1.7.D.1, 1.7.F.1, 1.7.F.2, 1.10.A.3, 1.10.D.1
3	2.A, 2.C, 2.E, 3.E, 4.E, 4.G	1, 3	1.2.A, 1.2.B, 3.12.A, 3.12.B, 3.12.C, 3.13.A, 3.13.C	1.2.A.2, 1.2.B.1, 3.12.A.1, 3.12.A.2, 3.12.B.1, 3.12.C.1, 3.13.A.1, 3.13.C.1, 3.13.C.2
4	2.C, 3.B, 3.C, 3.E, 4.B, 4.F, 4.G	1, 2, 4	1.7.F, 2.7.A, 2.9.A, 4.7.A, 4.7.C, 4.8.A, 4.8.B	1.7.F.2, 2.7.A.3, 2.9.A.2, 4.7.A.1, 4.7.C.2, 4.8.A.3, 4.8.B.1

The scoring information for the questions within this course and exam description, along with further exam resources, can be found on the [AP Statistics Exam page](#) on AP Central.

## AP STATISTICS

## Scoring Guidelines

## Question 1: Multi-Focus on Practices 1 and 2

10 points

## General Scoring Notes

- Except where otherwise noted, each point of these scoring guidelines is earned independently.
- The scoring criteria identify the specific components of the model solution that are used to determine the score.

A country's transportation department wants to conduct a survey to collect data from adults, ages 18 and older, in that country. The survey will be focused on commuting to work, including commute time in minutes, method of transportation used, and other information.

Country A is divided into many regions, and each region will be asked to send out a survey and collect the data. The transportation department will allow each region to use any method it preferred to survey the adults.

Three different regions will use the following sampling methods:

Region X: Each adult who lives in the region will receive a survey and will be required to complete the survey by mail. Adults who do not complete the survey will be sent reminders to do so until they have completed the survey.

Region Y: The region is divided into six areas that are similar to each other. Two of the six areas will be randomly selected, and each adult who lives in those two areas will receive a survey and be asked to complete the survey by mail. The table gives the results of the data collection for the two areas in Region Y for the question "What is your average daily commute time to work in minutes?"

Region Z: Each adult who lives in the region and has responded to a previous official survey will receive a survey and be asked to complete it by mail.

Summary Statistics

Area	$n$	$\bar{x}$	Min	Q1	Median (Q2)	Q3	Max
One	2,887	55	7	35	53	74	101
Five	4,502	42	5	24	32	68	83

Use the given information to respond to parts A, B, C, D, and E. Label any subparts (e.g., i and ii) that may be present.

A.

- Identify the sampling method for Region X.
- Identify the sampling method for Region Y.

B.

- Explain why the results of the sampling method for Region Y can be generalized to the population of the region.
- Tom, a member of the planning committee for Region Z, proposed Region Z's sampling method. Tom claims the sampling method for Region Z may lead to a higher survey response rate than the sampling methods used for the other two regions and could better represent the entire population in Region Z. Explain why Tom's claim is not correct.

- C. Manuel, an insurance agent in Region Y, wants to compare the average commute times to work for the populations in each of the two areas of Region Y. His first step is to construct a graphical representation for each area using the summary statistics in the table.
- Identify** a graphical representation that is appropriate to use to display the summary statistics.
  - Explain** why the graphical representation from part C (i) is appropriate to use.
- D. Use comparisons of the summary statistics in the table to **describe** the most likely shape for the distribution of commute time in Area Five.
- E. Manuel wants to perform a study about the average daily commute time to work for Area Three and Area Six in Region Y. He believes there is a difference in the commute times for the two areas. **Determine** a valid investigative question for Manuel's study. Ensure all three components of a valid investigative question are included in your response.

## Model Solution

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- A.
- i. The sampling method for Region X is a census.
  - ii. The sampling method for Region Y is cluster random sampling.
- B.
- i. The findings would represent the entire region since the six areas are similar to each other and data from two randomly selected areas are collected.
  - ii. Tom's sampling method is not better since it is not a random sample and also may not represent the population of the region because he is only surveying people who responded to a previous survey.
- C.
- i. A boxplot
  - ii. The table displays the five-number summary, which can be displayed with a boxplot.
- D. The mean (42) is larger than the median (32), which suggests that the distribution of commute time for Area Five is likely right-skewed.
- E. Is there convincing statistical evidence to suggest that the mean daily commute time to work for all adults (ages 18 and older) in Area Three of region Y is different than the mean daily commute time to work for all adults (ages 18 and older) in Area Six of region Y?
-

Reporting Category	Scoring Criteria	
<b>Part A (i)</b> <b>(0-1 points)</b> <b>Point 1</b>	<b>0 points</b> Does not accurately <b>identify</b> the relevant information	<b>1 point</b> Accurately <b>identifies</b> the relevant information
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately identify the sampling method in Region X.</li> <li>▪ The response does not provide identification for the sampling method in Region X.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately identifies the sampling method as a census for Region X.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "convenience sample"</li> <li>▪ "simple random sample"</li> <li>▪ "random sample"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "census"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part A (ii)</b> <b>(0-1 points)</b> <b>Point 2</b>	<b>0 points</b> Does not accurately <b>identify</b> the relevant information	<b>1 point</b> Accurately <b>identifies</b> the relevant information
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately identify the sampling method in Region Y.</li> <li>▪ The response does not provide identification for the sampling method in Region Y.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately identifies the sampling method as cluster random sampling for Region Y.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "random sample"</li> <li>▪ "stratified random sample"</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "cluster random sample"</li> <li>▪ "cluster random sampling"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part B (i)</b> <b>(0-1 points)</b> <b>Point 3</b>	<b>0 points</b> Does not accurately <b>explain</b> why a method is appropriate or not appropriate for gathering and representing data	<b>1 point</b> Accurately <b>explains</b> why a method is appropriate or not appropriate for gathering and representing data
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately explain why the results obtained using the sampling method in Region Y can be generalized to all registered voters in Region Y.</li> <li>▪ The response does not provide an explanation for why the results obtained using the sampling method in Region Y cannot be generalized to all registered voters in Region Y.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately explains that the method for Region Y is appropriate because the two areas were randomly selected and could represent the entire region because the six areas are similar to each other.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ “The results from Region Y cannot be generalized to all registered voters in Region Y.”</li> <li>▪ “The results from Region Y cannot be generalized to all registered voters in the region because only two areas were selected.”</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ “The findings for Region Y represent the entire region because the six areas are similar to each other and the two areas were randomly selected from the six similar areas.”</li> </ul>
	<b>Additional Notes:</b> <ul style="list-style-type: none"> <li>▪ The mention of bias is extraneous and can be ignored.</li> </ul>	

Reporting Category	Scoring Criteria	
<b>Part B (ii)</b> <b>(0-1 points)</b> <b>Point 4</b>	<b>0 points</b> Does not accurately <b>explain</b> why a method is appropriate or not appropriate for gathering and representing data	<b>1 point</b> Accurately <b>explains</b> why a method is appropriate or not appropriate for gathering and representing data
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately explain why the results obtained using Tom’s method cannot be generalized to all voters in the region.</li> <li>▪ The response does not provide an explanation that the results obtained using Tom’s plan cannot be generalized to all voters in the region and/or does not provide a reason.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response explains that Tom’s sampling method is not better because it is not a random sample and may not represent the population of the region.</li> <li>▪ The response explains that the survey was only sent to people who had previously responded to a survey and not randomly selected, so the results will not represent all the registered voters in that region.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ “The results can be generalized to all registered voters in the region.”</li> <li>▪ “The results will represent all the adults in that region.”</li> <li>▪ “The results cannot be generalized to all registered voters in the region.”</li> <li>▪ “The results will not represent all the adults in that region.”</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ “The results cannot be generalized to all registered voters in Region Z because the survey was not sent to people who were randomly selected and was only sent to people who previously responded to a survey, so the results will not represent all the registered voters in that region.”</li> <li>▪ “The adults who live in this region were not randomly selected, so the results will not represent all the registered voters in that region.”</li> </ul>
<b>Additional Notes:</b> <ul style="list-style-type: none"> <li>▪ The mention of bias is extraneous and can be ignored.</li> </ul>		

Reporting Category	Scoring Criteria	
<b>Part C (i)</b> <b>(0-1 points)</b> <b>Point 5</b>	<b>0 points</b> Does not accurately <b>identify</b> graphical representations of the data	<b>1 point</b> Accurately <b>identifies</b> graphical representations of the data
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response identifies an incorrect graphical representation.</li> <li>▪ The response does not provide a graphical representation.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response identifies a boxplot.</li> <li>▪ The response identifies a box-and-whisker plot.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "histogram"</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "boxplot"</li> <li>▪ "box-and-whisker plot"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C (ii)</b> <b>(0-1 points)</b> <b>Point 6</b>	<b>0 points</b> Does not accurately <b>explain</b> why graphical representations are appropriate to display the data	<b>1 point</b> Accurately <b>explains</b> why graphical representations are appropriate to display the data
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately explain why the graphical representation is appropriate to use.</li> <li>▪ The response does not provide an explanation for why the graphical representation is appropriate to use.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately explains that the table displays the five-number summary, which can be displayed with a boxplot.</li> <li>▪ A response that incorrectly identifies the graphical representation but gives a correct reason earns Part C Point 6 (but does not earn Part C Point 5).</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The data are quantitative."</li> <li>▪ "The data have the measures of center and spread."</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The table displays the five-number summary, which can be displayed with a boxplot."</li> <li>▪ "A table displays the min, Q1, median, Q3, and max, which can be displayed with a boxplot."</li> </ul>
	<b>Additional Notes:</b> <ul style="list-style-type: none"> <li>▪ The mention of bias is extraneous and can be ignored.</li> </ul>	

Reporting Category	Scoring Criteria	
<b>Part D</b> <b>(0-1 points)</b> <b>Point 7</b>	<b>0 points</b> Does not accurately <b>describe</b> numerical representations of data	<b>1 point</b> Accurately <b>describes</b> numerical representations of data
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response provides an incorrect description of the shape using the comparison of summary statistics.</li> <li>▪ The response does not provide a description of the shape using a comparison of the summary statistics.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response compares the median of 32 to the mean of 42 for Area Five and describes that the distribution is likely somewhat right-skewed.</li> <li>▪ The response indicates a comparison using the five-number summary and describes that the distribution is likely somewhat right-skewed.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The distribution is right-skewed."</li> <li>▪ "For Area Five, because the median (32) is smaller than the mean (42), the distribution is likely left-skewed."</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "For Area Five, because the median (32) is smaller than the mean (42), the distribution is likely right-skewed."</li> <li>▪ "For Area Five, the distance from Q1 to the median is smaller than the distance from the median to Q3, indicating that this distribution is likely right-skewed."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part E</b> <b>First Component</b> <b>(0-1 points)</b> <b>Point 8</b>	<b>0 points</b> Does not accurately <b>determine</b> the first component of an investigative question	<b>1 point</b> Accurately <b>determines</b> the first component of an investigative question
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately report the variables of interest.</li> <li>▪ The response does not provide variables of interest.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately reports the variables as the two areas and daily commute times.</li> <li>▪ The response accurately reports the variables as Areas Three and Six and daily commute times.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "areas"</li> <li>▪ "times"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "the two areas and daily commute times"</li> <li>▪ "the two areas and mean daily commute times"</li> <li>▪ "Areas Three and Six and average daily commute times"</li> <li>▪ "Areas Three and Six and daily commute times"</li> </ul>

Reporting Category	Scoring Criteria	
<p><b>Part E</b> <b>Second Component</b> <b>(0-1 points)</b> <b>Point 9</b></p>	<p><b>0 points</b> Does not accurately <b>determine</b> the second component of an investigative question</p>	<p><b>1 point</b> Accurately <b>determines</b> the second component of an investigative question</p>
	<p><b>Decision Rules and Scoring Notes</b></p>	
	<p><b>Responses that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response does not accurately report the population parameters or the direction of the alternative hypothesis.</li> <li>▪ The response does not provide the population parameters or the direction of the alternative hypothesis.</li> </ul>	<p><b>Responses that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response accurately reports the population parameters as the mean daily commute time for each area and the direction of alternative hypothesis as different (difference).</li> <li>▪ The response accurately reports the population parameters as the mean daily commute time for each area and the direction of alternative hypothesis as not the same.</li> </ul>
	<p><b>Examples that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "Daily commute times are equal."</li> <li>▪ "Mean daily commute times are equal."</li> <li>▪ "Daily commute time is greater than."</li> <li>▪ "Mean daily commute time is greater than."</li> <li>▪ "Daily commute time is less than."</li> <li>▪ "Mean daily commute time is less than."</li> </ul>	<p><b>Examples that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "Mean daily commute times for Areas Three and Six are different (difference)."</li> <li>▪ "Mean daily commute times for Areas Three and Six are not the same."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part E</b> <b>Third Component</b> <b>(0-1 points)</b> <b>Point 10</b>	<b>0 points</b> Does not accurately <b>determine</b> the third component of an investigative question	<b>1 point</b> Accurately <b>determines</b> the third component of an investigative question
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately report the population.</li> <li>▪ The response does not provide the population.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately reports that the population is all adults in Areas Three and Six of Region Y.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ “everyone in Region Y”</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ “all adults in Areas Three and Six of Region Y”</li> </ul>

## Question 2: Multi-Focus on Practices 3 and 4

10 points

## General Scoring Notes

- Except where otherwise noted, each point of these scoring guidelines is earned independently.
- The scoring criteria identify the specific components of the model solution that are used to determine the score.

Andrea, a cybersecurity specialist at a bank, was concerned about the security of the bank's website. She was asked to compare two encryption (a process where information is scrambled and cannot be easily read) methods that could be used to protect the website's banking information. Her department was concerned that the older encryption version, DES, may be weaker than the newer encryption version, AES. One measure of how well an encryption method works is session duration (how long the information is accessed), in seconds. A very long session duration may indicate that there has been unauthorized access to the data, which means that the encryption method needs to be more secure. Her department wanted her to study the two encryption methods.

Therefore, Andrea conducted a study of the two types of encryption methods. Andrea randomly collected this information from bank customers who used the website over the previous six months. For each session of the sampled customers, Andrea collected the session duration, in seconds, and the type of encryption method used. The table displays the summary statistics for her study.

Summary Statistics

Encryption Method	$n$	$\bar{x}$	$s$	Min	Q1	Median (Q2)	Q3	Max
DES	2,865	782	768	1	226	543	1,107	6,950
AES	4,706	794	803	1	233	794	1,091	7,190

Use the given information to respond to parts A, B, C, D, and E. Label any subparts (e.g., i and ii) that may be present.

- A.
- Identify** the type of study Andrea conducted.
  - Identify** the categorical variable in Andrea's study.
- B. Use the table to respond to the following. Show the work that leads to your answers.
- Determine** whether there are outliers using the  $1.5 \times \text{IQR}$  rule for the DES encryption method.
  - Determine** whether there are outliers using the  $1.5 \times \text{IQR}$  rule for the AES encryption method.
- C. Use the table to respond to the following.
- Compare** the distance from Q1 to the median and the distance from the median to Q3 for the DES encryption method.
  - Compare** the distance from Q1 to the median and the distance from the median to Q3 for the AES encryption method.
  - Explain** how these comparisons can be used to select which measure of variability—standard deviation, range, or IQR—would be the best representation for both of the encryption methods.
- D. Andrea decides to compare the medians of the two encryption methods to determine which encryption method is best. Use the table and your responses to part B and part C to **determine** whether Andrea is using the correct measure of center to compare the distributions. **Justify** your answer using both encryption methods.
- E. **Determine** the population for a valid investigative question for Andrea's study.

**Model Solution**

- A.
- Observational study
  - Encryption method
- B.
- For the DES method,  $1.5 \times \text{IQR}$  is  $1.5 \cdot (1,107 - 226) = 1,321.5$ . There are no lower outliers because  $226 - 1,321.5 = -1,095.5$  and the minimum number is 1. There are upper outliers because  $1,107 + 1,321.5 = 2,428.5$  and the maximum is 6,950.
  - For the AES method,  $1.5 \times \text{IQR}$  is  $1.5 \cdot (1,091 - 233) = 1,287$ . There are no lower outliers because  $233 - 1,287 = -1,054$  and the minimum number is 1. There are upper outliers because  $1,091 + 1,287 = 2,378$  and the maximum is 7,190.
- C.
- For the DES encryption method, the distance from Q1 to the median (317) is less than the distance from the median to Q3 (564).
  - For the AES encryption method, the distance from Q1 to the median (561) is greater than the distance from the median to Q3 (297).
  - The best measure of variability for both methods is the IQR because the distributions are not symmetric.
- D. Andrea is using the correct measure of center to compare the two encryption methods because both distributions had outliers, and within both distributions the distance from Q1 to the median and the median to Q3 were different lengths, indicating that neither distribution is symmetric.
- E. All the bank's customers who use the website
-

Reporting Category	Scoring Criteria	
<b>Part A (i)</b> <b>(0-1 points)</b> <b>Point 1</b>	<b>0 points</b> Does not accurately <b>identify</b> the relevant information	<b>1 point</b> Accurately <b>identifies</b> the relevant information
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately identify the type of study Andrea conducted.</li> <li>▪ The response does not provide the type of study Andrea conducted.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately identifies the type of study Andrea conducted as an observational study.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "experiment"</li> <li>▪ "random sample"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "observational study"</li> <li>▪ "retrospective observational study"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part A (ii)</b> <b>(0-1 points)</b> <b>Point 2</b>	<b>0 points</b> Does not accurately <b>identify</b> the relevant information	<b>1 point</b> Accurately <b>identifies</b> the relevant information
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately identify the variable.</li> <li>▪ The response does not provide the variable.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately classifies the categorical variable in Andrea's study as the encryption method.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "DES by itself"</li> <li>▪ "AES by itself"</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "encryption method"</li> <li>▪ "DES and AES"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part B (i)</b> <b>(0-1 points)</b> <b>Point 3</b>	<b>0 points</b> Does not accurately <b>determine</b> whether outliers exist	<b>1 point</b> Accurately <b>determines</b> whether outliers exist
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately determine the outliers using the <math>1.5 \times \text{IQR}</math> rule for the DES method.</li> <li>▪ The response does not accurately report that there are no lower outliers for the DES encryption methods.</li> <li>▪ The response does not accurately report that there are upper outliers for the DES encryption method.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately determines that there are outliers using the <math>1.5 \times \text{IQR}</math> rule for the DES method and provides supporting work.</li> <li>▪ An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ “<math>1.5 \times \text{IQR}</math>, so there are no lower outliers but upper outliers for DES.” (No work is shown.)</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ “For the DES method, <math>1.5 \times \text{IQR}</math> is <math>1.5 \cdot (1,107 - 226) = 1,321.5</math>. There are no lower outliers because <math>226 - 1,321.5 = -1,095.5</math> and the minimum number is 1. There are upper outliers because <math>1,107 + 1,321.5 = 2,428.5</math> and the maximum is 6,950.”</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part B (ii)</b> <b>(0-1 points)</b> <b>Point 4</b>	<b>0 points</b> Does not accurately <b>determine</b> whether outliers exist	<b>1 point</b> Accurately <b>determines</b> whether outliers exist
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately determine the <math>1.5 \times \text{IQR}</math> method for the AES method.</li> <li>▪ The response does not accurately report that there are no lower outliers for the AES encryption method.</li> <li>▪ The response does not accurately report that there are upper outliers for the AES encryption method.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately determines that there are outliers using the <math>1.5 \times \text{IQR}</math> rule for the AES method and provides supporting work.</li> <li>▪ An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "<math>1.5 \times \text{IQR}</math>, so there are no lower outliers but upper outliers for both methods." (No work is shown.)</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "For the AES method, <math>1.5 \times \text{IQR}</math> is <math>1.5 \cdot (1,091 - 233) = 1,287</math>. There are no lower outliers because <math>233 - 1,287 = -1,054</math> and the minimum number is 1. There are upper outliers because <math>1,091 + 1,287 = 2,378</math> and the maximum is 7,190."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C (i)</b> <b>(0-1 points)</b> <b>Point 5</b>	<b>0 points</b> Does not accurately <b>compare</b> numerical representations of data	<b>1 point</b> Accurately <b>compares</b> numerical representations of data
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response fails to compare the distance from Q1 to the median and the distance from the median to Q3.</li> <li>▪ The response gives incorrect values but does not have any work shown to indicate that this was just a minor mathematical error.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response indicates that the distance from Q1 to the median is less than the distance from the median to Q3.</li> <li>▪ The response indicates that the distance from Q1 to the median is less than the distance from the median to Q3 but gives incorrect values. However, the work shown indicates that this is a minor mathematical error.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The distance from Q1 to the median is 317, and the distance from the median to Q3 is 564."</li> <li>▪ "The distance from Q1 to the median is 217, which is less than the distance from the median to Q3 (564)." (No work is shown to verify that this was a minor mathematical error.)</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "For the DES encryption method, the distance from Q1 to the median (317) is less than the distance from the median to Q3 (564)."</li> <li>▪ "For the DES encryption method, the distance from Q1 to the median is less than the distance from the median to Q3."</li> <li>▪ "For the DES encryption method, the distance from Q1 to the median (<math>543 - 226 = 217</math>) is less than the distance from the median to Q3 (<math>1,107 - 543 = 564</math>)." (Work is shown, and 217 is treated as a minor mathematical error.)</li> </ul>	

Reporting Category	Scoring Criteria	
<b>Part C (ii)</b> <b>(0-1 points)</b> <b>Point 6</b>	<b>0 points</b> Does not accurately <b>compare</b> numerical representations of data	<b>1 point</b> Accurately <b>compares</b> numerical representations of data
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response fails to compare the distance from Q1 to the median and the distance from the median to Q3.</li> <li>▪ The response gives incorrect values but does not have any work shown to indicate that this was just a minor mathematical error.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response indicates that the distance from Q1 to the median is greater than the distance from the median to Q3.</li> <li>▪ The response indicates that the distance from Q1 to the median is greater than the distance from the median to Q3 but gives incorrect values. However, the work shown indicates that this is a minor mathematical error.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The distance from Q1 to the median is 561, and the distance from the median to Q3 is 297."</li> <li>▪ "The distance from Q1 to the median is 461, and the distance from the median to Q3 is 297." (No work is shown to verify that this was a minor mathematical error.)</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "For the AES encryption method, the distance from Q1 to the median (561) is larger than the distance from the median to Q3 (297)."</li> <li>▪ "For the AES encryption method, the distance from Q1 to the median is larger than the distance from the median to Q3."</li> <li>▪ "For the AES encryption method, the distance from Q1 to the median (<math>794 - 233 = 461</math>) is larger than the distance from the median to Q3 (<math>1,091 - 794 = 297</math>)." (Work is shown, and 461 is treated as a minor mathematical error.)</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C (iii)</b> <b>(0-1 points)</b> <b>Point 7</b>	<b>0 points</b> Does not accurately <b>explain</b> numerical representations of data	<b>1 point</b> Accurately <b>explains</b> numerical representations of data
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately explain that the IQR is the best measure of variability for the DES method.</li> <li>▪ The response does not accurately link the comparison of the distances to the shape of the distribution, which will affect the selection of the measure of variability.</li> <li>▪ The response does not give a reason to use the IQR.</li> <li>▪ The response does not refer to the comparisons.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response uses the comparison of the distances from Q1 to the median and the median to Q3 to indicate that the IQR is the best measure of variability because it is evident that neither method is symmetric.</li> <li>▪ The response shows evidence that neither method is symmetric, which will affect the selection of the measure of variability.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The best measure of variability to represent both methods is the IQR."</li> <li>▪ "Based on these comparisons, the range is the best measure of variability to represent both methods."</li> <li>▪ "Based on these comparisons, the standard deviation is the best measure of variability to represent both methods."</li> <li>▪ "The shape of the distribution for both methods is fairly symmetric and therefore will need to be considered when Andrea chooses a measure of variability for both methods."</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Because the distance from Q1 to the median and the distance from the median to Q3 are very different within each distribution, the distributions are likely skewed. Therefore, the IQR is the best measure of variability for both methods."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part D</b> <b>(0-1 points)</b> <b>Point 8</b>	<b>0 points</b> Does not accurately <b>determine</b> whether a claim is supported or not supported based on statistical results	<b>1 point</b> Accurately <b>determines</b> whether a claim is supported or not supported based on statistical results
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately determine that Andrea is using the best measure of center.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately determines that Andrea is using the best measure of center.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Neither is correct."</li> <li>▪ "Both measures of center can be used."</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Andrea is correct in using the medians to compare the encryption methods."</li> </ul>

Reporting Category	Scoring Criteria	
<p><b>Part D</b> <b>(0-1 points)</b> <b>Point 9</b></p>	<p><b>0 points</b> Does not accurately <b>justify</b> the answer based on statistical results</p>	<p><b>1 point</b> Accurately <b>justifies</b> the answer based on statistical results</p>
<b>Decision Rules and Scoring Notes</b>		
<p><b>Responses that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response does not accurately justify the answer using other parts of the question.</li> <li>▪ The response does not justify the answer using both encryption methods.</li> <li>▪ The response does not discuss that medians are resistant to outliers.</li> </ul>		<p><b>Responses that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response accurately justifies Andrea’s use of the medians by noting that both distributions have outliers and/or that the distance from Q1 to the median is very different than the distance from the median to Q3 within each distribution, indicating that the distributions are not symmetric. Therefore, the medians are better because they are resistant to outliers, and the means are not resistant to outliers.</li> </ul>
<p><b>Examples that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ “proof from part B”</li> <li>▪ “The mean and the median are different.”</li> <li>▪ “The mean and the median are the same.”</li> <li>▪ The medians are better.”</li> </ul>		<p><b>Examples that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ “Both distributions have outliers, which may indicate that the distributions are not symmetric. Therefore, the medians are better because they are resistant to outliers, and the means are not resistant to outliers.”</li> <li>▪ “The distance from Q1 to the median is not the same as the distance from the median to Q3 in either distribution, which may indicate that the distributions are not symmetric. Therefore, the medians are better because they are resistant to outliers, and the means are not resistant to outliers.”</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part E</b> <b>(0-1 points)</b> <b>Point 10</b>	<b>0 points</b> Does not accurately <b>determine</b> the indicated component for a valid investigative question	<b>1 point</b> Accurately <b>determines</b> the indicated component for a valid investigative question
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately determine the population.</li> <li>▪ The response does not provide the population.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately determines that the population for a valid investigative question for Andrea's study is all the bank's customers who use the bank's website.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "all customers"</li> <li>▪ "all bank customers"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "all the bank's customers who use the bank's website"</li> </ul>

**Question 3: Inference (Hypothesis Test)****10 points****General Scoring Notes**

- Except where otherwise noted, each point of these scoring guidelines is earned independently.
- The scoring criteria identify the specific components of the model solution that are used to determine the score.

Neil, a marketing director, wanted to compare the effectiveness of two different video advertisements designed to persuade people to buy a new laptop. He showed the two advertisements, version A and version B, to different groups of people. The responses from the different groups of people were compared to determine which version performs better. Each advertisement contains a unique code that people can text using their cell phones to get more information about the laptop. Neil will use the advertisement that has a higher percentage of people who text that version's code. If there is no difference in the percentages for the two advertisements, he will use both.

Neil gathered a group of 500 volunteers who were interested in purchasing a laptop. The volunteers were randomly assigned to groups. One group watched version A of the advertisement for the new laptop, and the other group watched version B of the advertisement for the new laptop. For version A, 83 of the 250 volunteers texted the code to get more information. For version B, 54 of the 250 volunteers texted the code to get more information.

At the 0.05 level of significance, complete the appropriate inference procedure to determine if there is convincing statistical evidence of a difference in the proportion of adults similar to those in the study who will watch version A and text the code than the proportion of adults similar to those in the study who will watch version B and text the code.

Use the given information to respond to parts A, B, C, and D.

- Identify** and **classify** the variable(s) in the study.
- Identify** the appropriate inference procedure.
- Complete** the inference procedure, including **calculating** the appropriate statistics.
- Justify** a conclusion in context.

## Model Solution

- A. The explanatory variable is the version of the commercial viewed and was categorical with categories A and B. The response variable in this study is whether a viewer texts for more information and was categorical: Yes or No.
- B. An appropriate inference procedure is a two-sample  $z$ -test for the difference between two population proportions.
- C. The null hypothesis is  $H_0: p_A = p_B$ , and the alternative hypothesis is  $H_a: p_A \neq p_B$ . For adults similar to those in the study, let  $p_A$  represent the true proportion who will watch version A and text the code for more information, and let  $p_B$  represent the true proportion who will watch version B and text the code for more information. The randomization condition for performing a two-sample  $z$ -test for the difference between two population proportions is satisfied because the volunteers were randomly assigned to Version A or Version B. The values of the sample

proportions are  $\hat{p}_A = \frac{83}{250} = 0.332$ ,  $\hat{p}_B = \frac{54}{250} = 0.216$ . The combined proportion

is  $\hat{p}_c = \frac{250(0.332) + 250(0.216)}{250 + 250} = 0.274$ . The normality condition is met because the expected number of successes and failures for both samples are at least 10.

Version A:  $250(0.274) = 68.5$ ,  $250(1 - 0.274) = 181.5$

Version B:  $250(0.274) = 68.5$ ,  $250(1 - 0.274) = 181.5$

The value of the test statistic is  $z = \frac{0.332 - 0.216}{\sqrt{0.274(1 - 0.274)} \sqrt{\frac{1}{250} + \frac{1}{250}}} = 2.91$ , and the  $p$ -value is 0.0036.

- D. Because this  $p$ -value of 0.0036 is less than  $\alpha = 0.05$  significance level, the null hypothesis should be rejected. There is convincing statistical evidence to conclude that there is a difference in the proportion of adults similar to those in the study who will watch version A and text the code for more information and the proportion of adults similar to those in the study who will watch version B and text the code for more information.

Reporting Category	Scoring Criteria	
<p><b>Part A</b> <b>Identify Variable</b> <b>(0-1 points)</b> <b>Point 1</b></p>	<p><b>0 points</b> Does not accurately <b>identify</b> the relevant information</p>	<p><b>1 point</b> Accurately <b>identifies</b> the relevant information</p>
	<p><b>Decision Rules and Scoring Notes</b></p>	
	<p><b>Responses that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response does not accurately identify the variables used in the study.</li> <li>▪ The response does not provide identification for the variables used in the study.</li> </ul>	<p><b>Responses that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response accurately identifies the explanatory variable as the version of the commercial viewed and the response variable as whether a viewer texts for more information.</li> </ul>
	<p><b>Examples that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "laptop"</li> <li>▪ "A and B"</li> </ul>	<p><b>Examples that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "version of the commercial and whether a person texts a code"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part A</b> <b>Classify Variable</b> <b>(0-1 points)</b> <b>Point 2</b>	<b>0 points</b> Does not accurately <b>classify</b> the relevant information	<b>1 point</b> Accurately <b>classifies</b> the relevant information
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately classify the variables used in the study as categorical or qualitative.</li> <li>▪ The response does not provide classification for the variables used in the study.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately classifies the variables used in the study as categorical or qualitative.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Both are quantitative."</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Both are qualitative."</li> <li>▪ "Both are categorical."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part B</b> <b>Identify Procedure</b> <b>(0-1 points)</b> <b>Point 3</b>	<b>0 points</b> Does not accurately <b>identify</b> the appropriate inference procedure	<b>1 point</b> Accurately <b>identifies</b> the appropriate inference procedure
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response identifies the incorrect inference procedure.</li> <li>▪ The response does not accurately name the correct inference procedure.</li> <li>▪ The response identifies the correct test by name but also includes an incorrect formula.</li> <li>▪ The response identifies the test by formula using a <math>t</math>-statistic instead of a <math>z</math>-test statistic.</li> <li>▪ The response does not provide an inference procedure name or formula.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately describes the inference procedure as a two-sample <math>z</math>-test for the difference between two population proportions.</li> <li>▪ The response accurately provides the formula for a two-sample <math>z</math>-test for the difference between two population proportions.               <math display="block">z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}_c(1 - \hat{p}_c)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}</math> </li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "test for two population proportions"</li> <li>▪ "two-proportion <math>z</math>-interval"</li> <li>▪ "2 <math>z</math>-test"</li> <li>▪ "two-sample <math>z</math>-test"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "two sample <math>z</math>-test for population proportions"</li> <li>▪ "two-sample <math>z</math>-test difference between two population proportions"               <math display="block">z = \frac{(\hat{p}_1 - \hat{p}_2) - 0}{\sqrt{\hat{p}_c(1 - \hat{p}_c)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} "</math> </li> </ul>

Reporting Category	Scoring Criteria	
<p><b>Part C</b> <b>Hypotheses</b> <b>(0-1 points)</b> <b>Point 4</b></p>	<p><b>0 points</b> Does not accurately <b>identify</b> the null and alternative hypotheses</p>	<p><b>1 point</b> Accurately <b>identifies</b> the null and alternative hypotheses</p>
	<p><b>Decision Rules and Scoring Notes</b></p>	
	<p><b>Responses that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response identifies the incorrect null and/or alternative hypotheses.</li> <li>▪ The response is not stated in terms of the proportions.</li> <li>▪ The response only provides one hypothesis.</li> <li>▪ The response does not provide both hypotheses.</li> </ul>	<p><b>Responses that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response identifies the null hypothesis as <math>H_0: p_A = p_B</math> and the alternative hypothesis as <math>H_a: p_A \neq p_B</math>.</li> </ul>
	<p><b>Examples that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "<math>H_0: p_A \neq p_B</math> and <math>H_a: p_A = p_B</math>"</li> <li>▪ "The null hypothesis is that the proportion subscript A does not equal the proportion subscript B, and the proportion subscript A equals the proportion subscript B."</li> </ul>	<p><b>Examples that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "<math>H_0: p_A = p_B</math> and <math>H_a: p_A \neq p_B</math>"</li> <li>▪ "The null hypothesis is that the proportion subscript A equals the proportion subscript B, and the alternative hypothesis is that the proportion subscript A does not equal the proportion subscript B."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C</b> <b>Parameter or Population</b> <b>(0-1 points)</b> <b>Point 5</b>	<b>0 points</b> Does not accurately <b>identify</b> the population parameter(s)	<b>1 point</b> Accurately <b>identifies</b> the population parameter(s)
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response identifies the incorrect parameters.</li> <li>▪ The response is stated in terms of the sample proportions.</li> <li>▪ The response does not provide parameters.</li> <li>▪ The response clearly refers to the sample proportions instead of the population proportions using words or a symbol (e.g., <math>\hat{p}</math>).</li> </ul> <b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The parameter, <math>\hat{p}</math>, represents the proportion."</li> <li>▪ "The parameter, <math>p</math>, represents the sample proportion."</li> <li>▪ "Let <math>\hat{p}_A</math> represent the proportion of people who watched version A and texted the code for more information, and let <math>\hat{p}_B</math> represent the proportion of people who watched version B and texted the code for more information."</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response identifies the correct parameters in terms of the population proportions.</li> <li>▪ The response that does not clearly indicate a population proportion in the parameter identification may satisfy the population aspect by clearly referring to the population in the justification (Part D Point 10).</li> </ul> <b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "For adults similar to those in the study, let <math>p_A</math> represent the population proportion of adults who will watch version A and text the code for more information, and let <math>p_B</math> represent the population proportion of adults who will watch version B and text the code for more information."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C</b> <b>Randomization and 10% Conditions</b> <b>(0-1 points)</b> <b>Point 6</b>	<b>0 points</b> Does not accurately <b>justify</b> whether the randomization condition and 10% condition are met	<b>1 point</b> Accurately <b>justifies</b> whether the randomization condition and 10% condition are met
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response only responds with random check.</li> <li>▪ The response implies that random assignment was used.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately justifies that the randomization condition is met because volunteers were randomly assigned to version A or version B.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "random check"</li> <li>▪ "random assignment"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The volunteers were randomly assigned to version A or version B."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C</b> <b>Normality or Expected Counts Conditions</b> <b>(0-1 points)</b> <b>Point 7</b>	<b>0 points</b> Does not accurately <b>justify</b> whether the normality condition is met	<b>1 point</b> Accurately <b>justifies</b> whether the normality condition is met
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response checks any inappropriate condition, such as <math>n &gt; 30</math>.</li> <li>▪ The response states that, according to the central limit theorem (CLT), the sampling distribution is approximately normal.</li> <li>▪ The response does not provide the normality condition.</li> </ul> <b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Because <math>np \geq 5</math> and <math>n(1 - p) \geq 5</math>, the sampling distribution is approximately normal."</li> <li>▪ "Because <math>np \geq 10</math> and <math>n(1 - p) \geq 10</math>, the sampling distribution is approximately normal."</li> <li>▪ "<math>n &gt; 30</math>"</li> <li>▪ "According to the central limit theorem (CLT), the sampling distribution is approximately normal."</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately justifies that the normality condition is met using a pooled <math>p</math>-hat because expected successes and failures for both samples are at least 10. Version A: <math>250(0.274) = 68.5</math>, <math>250(1 - 0.274) = 181.5</math> Version B: <math>250(0.274) = 68.5</math>, <math>250(1 - 0.274) = 181.5</math></li> <li>▪ The response accurately justifies that the normality condition is met using a non-pooled <math>p</math>-hat because observed successes and failures for both samples are at least 10. Version A: <math>250(0.332) = 83</math>, <math>250(1 - 0.332) = 167</math> Version B: <math>250(0.216) = 54</math>, <math>250(1 - 0.216) = 196</math></li> </ul> <b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The normality condition is met because the expected number of successes and failures for both samples are at least 10. <math>250(0.274) = 68.5</math>, <math>250(1 - 0.274) = 181.5</math>."</li> <li>▪ "The normality condition is met because the observed number of successes and failures for both samples are at least 10. Version A: <math>250(0.332) = 83</math>, <math>250(1 - 0.332) = 167</math> Version B: <math>250(0.216) = 54</math>, <math>250(1 - 0.216) = 196</math>"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C</b> <b>Test Statistics and <math>p</math>-Value</b> <b>(0-1 points)</b> <b>Point 8</b>	<b>0 points</b> Does not accurately <b>calculate</b> the results for the appropriate statistical inference method	<b>1 point</b> Accurately <b>calculates</b> the results for the appropriate statistical inference method
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately calculate the <math>z</math>-statistic and/or <math>p</math>-value, and supporting work is not shown to indicate a minor calculation error.</li> <li>▪ The response does not provide the <math>z</math>-statistic and/or <math>p</math>-value.</li> </ul> <b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The <math>z</math>-statistic is 3, and the <math>p</math>-value is less than 1."</li> <li>▪ "The <math>z</math>-statistic is 2.89, and the <math>p</math>-value is 0."</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately reports the <math>z</math>-statistic as 2.91 and the <math>p</math>-value as 0.0036.</li> <li>▪ Supporting work for the correct <math>p</math>-value is considered extraneous and can be ignored.</li> <li>▪ An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> </ul> <b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The <math>z</math>-statistic is 2.91, and the <math>p</math>-value is 0.0036."</li> <li>▪ "The value of the test statistic is               <math display="block">z = \frac{0.332 - 0.216}{\sqrt{0.27(1-0.27)}\sqrt{\frac{1}{250} + \frac{1}{250}}} = 2.91</math>               and the <math>p</math>-value is 0.0036."             </li> <li>▪ "The value of the test statistic is               <math display="block">z = \frac{0.332 - 0.216}{\sqrt{\frac{(0.332)(1-0.332)}{250} + \frac{(0.216)(1-0.216)}{250}}} = 2.93</math>               and the <math>p</math>-value is 0.0034."             </li> </ul>

Reporting Category	Scoring Criteria	
<b>Part D</b> <b><math>p</math>-Value to the Level of Significance Comparison</b> <b>(0-1 points)</b> <b>Point 9</b>	<b>0 points</b> Does not accurately <b>determine</b> the results from a comparison of the $p$ -value to the level of significance	<b>1 point</b> Accurately <b>determines</b> the results from a comparison of the $p$ -value to the level of significance
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately compare the <math>p</math>-value to the level of significance.</li> <li>▪ The response does not include values for the <math>p</math>-value and the level of significance.</li> <li>▪ The response does not accurately conclude the formal decision about the null hypothesis based on the comparison of the <math>p</math>-value to the level of significance.</li> <li>▪ The response includes an incorrect <math>p</math>-value for the comparison.</li> <li>▪ The response does not compare the <math>p</math>-value to the level of significance.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response compares the correct <math>p</math>-value and the level of significance with a formal decision about the null hypothesis.</li> <li>▪ The response compares the correct <math>p</math>-value with a formal decision about the null hypothesis and the incorrect, consistently used level of significance.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "<math>p</math> is less than alpha."</li> <li>▪ "<math>0.0036 &gt; 0.05</math>"</li> <li>▪ "<math>0.0036 &gt; 0.05</math>, fail to reject the null hypothesis"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The <math>p</math>-value of 0.0036 is <math>&lt;</math> alpha of 0.05, so the null hypothesis should be rejected."</li> <li>▪ "<math>0.0036 &lt; 0.05</math>. Therefore, the null should be rejected."</li> <li>▪ "Because the <math>p</math>-value (0.0036) is less than the level of significance (0.05), reject <math>H_0</math>."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part D</b> <b>Justification</b> <b>(0-1 points)</b> <b>Point 10</b>	<b>0 points</b> Does not accurately <b>justify</b> whether a claim is supported or not supported based on statistical calculations and results	<b>1 point</b> Accurately <b>justifies</b> whether a claim is supported or not supported based on statistical calculations and results
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not include wording that there is supporting evidence.</li> <li>▪ The response does not include the alternative hypothesis.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response indicates there is enough statistical evidence to conclude that there is a difference in the proportion of adults similar to those in the study who will watch version A and text the code for more information and the proportion of adults similar to those in the study who will watch version B and text the code for more information.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ “There is no difference in the percentage of people who watched version A and texted the code for more information and the percentage of people who watched version B and texted the code for more information.”</li> <li>▪ “The test supports Neil’s claim.”</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ “There is convincing statistical evidence of a difference in the proportion of adults similar to those in the study who will watch version A and text the code for more information and the proportion of adults similar to those in the study who will watch version B and text the code for more information.”</li> </ul>

## Question 4: Multi-Focus on Practices 2, 3, and 4

10 points

## General Scoring Notes

- Except where otherwise noted, each point of these scoring guidelines is earned independently.
- The scoring criteria identify the specific components of the model solution that are used to determine the score.

According to a 2024 national survey of 400 randomly selected offices in a certain country, the average number of water bottle refilling stations was 2.6 with a standard deviation of 1.6 stations. Erika is a marketer at a water bottle company, and she wants to investigate whether the average number of refilling stations per office has changed from 2024 to 2025. In 2025, she selected a random sample of 350 offices in the country and asked how many refilling stations were in each office. The results of the 2025 sample are summarized in the table.

Number of Water Bottle Refilling Stations and Proportions of Offices in 2025

Number of Refilling Stations	0	1	2	3	4	5	6
Proportion of Offices	0.04	0.13	0.21	0.35	0.19	0.05	0.03

Erika also found the standard deviation for the sample to be 1.31 refilling stations.

Use the given information to respond to parts A, B, C, D, and E. Label any subparts (e.g., i and ii) that may be present.

- A. Use the table to respond to the following. Show all work for calculations.
- An office from the sample will be selected at random. **Calculate** the probability that an office had 3 or more refilling stations in 2025.
  - Calculate** the average number of refilling stations per office for the sample of offices in 2025.
- B. Erika believes that the average number of refilling stations per office for the sample of offices in 2025 has changed compared to the sample of offices from 2024. **Determine** whether Erika's belief is supported by the data. Use the results from part A (ii) to **justify** your answer without conducting an inference procedure.
- C. Erika wants to conduct an inference procedure to estimate the difference in the true mean number of refilling stations per office between 2024 and 2025. **Identify** the appropriate inference procedure Erika should use.
- D. Assume the conditions for the inference procedure identified in part C were satisfied and that Erika used a 90% confidence level.
- Calculate** the results from the appropriate inference procedure.
  - Interpret** a conclusion from part D (i) in context.
- E. Based on the results from Erika's inference procedure, **determine** whether there is sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025. **Explain** your reasoning.

**Model Solution**

- A.
- $P(X \geq 3) = 0.35 + 0.19 + 0.05 + 0.03 = 0.62$
  - $E(X) = (0 \cdot 0.04) + (1 \cdot 0.13) + (2 \cdot 0.21) + (3 \cdot 0.35) + (4 \cdot 0.19) + (5 \cdot 0.05) + (6 \cdot 0.03) = 2.79$  refilling stations
- B. There has been a change in the average number of refilling stations per office in the country, from 2.6 refilling stations in 2024 to 2.79 refilling stations in 2025.
- C. A two-sample  $t$ -interval for a difference between two population means
- D. Assume the conditions for inference were met for the inference procedure identified in part D.
- The 90% confidence interval is  $(-0.365, -0.015)$ .
  - Erika is 90% confident that the difference in the true mean number of refilling stations per office between 2024 and 2025 falls between  $-0.365$  and  $-0.015$ .
- E. Based on Erika's inference procedure, there is sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025 because 0 is not in the interval.
-

Reporting Category	Scoring Criteria	
<b>Part A (i)</b> <b>(0-1 points)</b> <b>Point 1</b>	<b>0 points</b> Does not accurately <b>calculate</b> the value	<b>1 point</b> Accurately <b>calculates</b> the value
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately calculate the proportion.</li> <li>▪ The response does not provide the proportion.</li> <li>▪ The response states a value of 0.62 but does not show the calculation.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately calculates <math>P(X \geq 3) = 0.35 + 0.19 + 0.05 + 0.03 = 0.62</math>.</li> <li>▪ An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "0.62"</li> <li>▪ "<math>0.19 + 0.05 + 0.03 = 0.27</math>"</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "<math>0.35 + 0.19 + 0.05 + 0.03 = 0.62</math>"</li> <li>▪ "<math>0.35 + 0.19 + 0.05 + 0.03 = 0.61</math>"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part A (ii)</b> <b>(0-1 points)</b>	<b>0 points</b> Does not accurately <b>calculate</b> the value	<b>1 point</b> Accurately <b>calculates</b> the value
<b>Point 2</b>	<b>Decision Rules and Scoring Notes</b>	
	<p><b>Responses that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response does not accurately calculate the value.</li> <li>▪ The response does not provide the value.</li> <li>▪ The response states a value of 2.79 but does not show the calculation.</li> </ul>	<p><b>Responses that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response accurately calculates <math>E(X) = (0 \cdot 0.04) + (1 \cdot 0.13) + (2 \cdot 0.21) + (3 \cdot 0.35) + (4 \cdot 0.19) + (5 \cdot 0.05) + (6 \cdot 0.03) = 2.79</math>.</li> <li>▪ An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> </ul>
	<p><b>Examples that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "2.79"</li> </ul>	<p><b>Examples that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "<math>(0 \cdot 0.04) + (1 \cdot 0.13) + (2 \cdot 0.21) + (3 \cdot 0.35) + (4 \cdot 0.19) + (5 \cdot 0.05) + (6 \cdot 0.03) = 2.79</math>"</li> <li>▪ "<math>(0 \cdot 0.04) + (1 \cdot 0.13) + (2 \cdot 0.21) + (3 \cdot 0.35) + (4 \cdot 0.19) + (5 \cdot 0.05) + (6 \cdot 0.03) = 2.69</math>"</li> </ul>

Reporting Category	Scoring Criteria	
<p><b>Part B</b> <b>(0-1 points)</b> <b>Point 3</b></p>	<p><b>0 points</b> Does not accurately <b>determine</b> whether a claim is supported or not supported based on statistical calculations</p>	<p><b>1 point</b> Accurately <b>determines</b> whether a claim is supported or not supported based on statistical calculations</p>
<b>Decision Rules and Scoring Notes</b>		
<p><b>Responses that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response does not indicate that there has been a change in the average number of refilling stations.</li> <li>▪ The response does not indicate that Erika's belief is supported.</li> </ul>		<p><b>Responses that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ The response indicates that there has been a change in the average number of refilling stations.</li> <li>▪ The response indicates that Erika's belief is supported.</li> </ul>
<p><b>Examples that do not earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "There is no change in the average number of refilling stations."</li> <li>▪ "There is a decrease in the average number of refilling stations."</li> </ul>		<p><b>Examples that earn this point:</b></p> <ul style="list-style-type: none"> <li>▪ "There has been a change in the average number of refilling stations per office between 2024 and 2025."</li> <li>▪ "Yes, Erika's belief is supported by the data."</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part B</b> <b>(0-1 points)</b> <b>Point 4</b>	<b>0 points</b> Does not accurately <b>justify</b> the answer based on statistical calculations	<b>1 point</b> Accurately <b>justifies</b> the answer based on statistical calculations
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not provide a correct explanation and/or justification with support.</li> <li>▪ The response does not provide any explanation and/or justification with support.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response indicates that the average number of refilling stations per office was 2.6 in 2024, which differs from 2.79 in 2025.</li> <li>▪ The response includes 2.79 or the value from part A (ii).</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "There is no change in the average number of refilling stations."</li> <li>▪ "There is a decrease in the average number of refilling stations."</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "In 2024, there was an average of 2.6 refilling stations, which differs from an average of 2.79 refilling stations in 2025."</li> <li>▪ "<math>2.79 &gt; 2.6</math>"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part C</b> <b>(0-1 points)</b> <b>Point 5</b>	<b>0 points</b> Does not accurately <b>identify</b> the appropriate inference procedure	<b>1 point</b> Accurately <b>identifies</b> the appropriate inference procedure
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response identifies the incorrect inference procedure.</li> <li>▪ The response does not accurately name the correct inference procedure.</li> <li>▪ The response identifies the correct inference procedure by name but also includes an incorrect formula.</li> <li>▪ The response does not provide an inference procedure name or formula.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately describes the inference procedure as a two-sample <math>t</math>-interval for a difference between two population means.</li> <li>▪ The response accurately provides the formula for a two-sample <math>t</math>-interval for a difference between two population means.               <math display="block">(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}</math> </li> <li>▪ <math>(2.6 - 2.79) \pm 1.645 \sqrt{\frac{1.6^2}{400} + \frac{1.31^2}{350}}</math></li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "A one-sample <math>t</math>-test for the population mean difference"</li> <li>▪ "A two-sample <math>z</math>-interval for a difference between two population means"</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "A two-sample <math>t</math>-interval for a difference between two population means"</li> <li>▪ "<math>(\bar{x}_1 - \bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}</math>"</li> <li>▪ "<math>(2.6 - 2.79) \pm 1.645 \sqrt{\frac{1.6^2}{400} + \frac{1.31^2}{350}}</math>"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part D (i)</b> <b>(0-1 points)</b> <b>Point 6</b>	<b>0 points</b> Does not accurately <b>calculate</b> the results for the appropriate statistical inference method	<b>1 point</b> Accurately <b>calculates</b> the results for the appropriate statistical inference method
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately calculate the <math>t</math>-interval for a difference between two population means.</li> <li>▪ The response does not provide a <math>t</math>-interval for a difference between two population means.</li> <li>▪ The response provides the correct interval but specifies it as a <math>z</math>-interval (unless the test named in part C was a two-sample <math>z</math>-interval).</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately reports the correct values for the <math>t</math>-interval for a difference between two population means as <math>(-0.365, -0.015)</math>.</li> <li>▪ <math>(2.6 - 2.79) \pm 1.645 \sqrt{\frac{1.6^2}{400} + \frac{1.31^2}{350}}</math>  <math>= (-0.365, -0.015)</math>,            with <math>df = 744.782</math>.</li> <li>▪ The response has a slightly different interval because the degrees of freedom used was more conservative, such as <math>df = 349</math>.</li> <li>▪ An arithmetic or transcription error in a response can be ignored if correct work is shown.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The <math>z</math>-interval is <math>(-0.365, -0.015)</math>."</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The <math>t</math>-interval is <math>(-0.365, -0.015)</math>, with <math>df = 744.782</math>."</li> <li>▪ "The <math>t</math>-interval is <math>(-0.367, -0.013)</math>, with <math>df = 349</math>."</li> </ul>
<b>Additional Notes:</b> <ul style="list-style-type: none"> <li>▪ The response is consistent with the stated inference procedure identified in part C.</li> </ul>		

Reporting Category	Scoring Criteria	
<b>Part D (ii)</b> <b>(0-1 points)</b> <b>Point 7</b>	<b>0 points</b> Does not accurately <b>interpret</b> the results for the appropriate statistical inference method	<b>1 point</b> Accurately <b>interprets</b> the results for the appropriate statistical inference method
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The interpretation does not include the 90% confidence level and the interval endpoints <math>(-0.365, -0.015)</math>.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The interpretation includes the 90% confidence level and the interval endpoints <math>(-0.365, -0.015)</math>.</li> <li>▪ The interpretation includes the confidence level and the interval endpoints consistent with the response from part D (i).</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "The interval is <math>(-0.365, -0.015)</math>."</li> <li>▪ "90% confidence level"</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "90% confident, and the interval is <math>(-0.365, -0.015)</math>."</li> </ul>
<b>Additional Notes:</b> <ul style="list-style-type: none"> <li>▪ Part D Points 7 and 8 can be earned in any order based on the response.</li> </ul>		

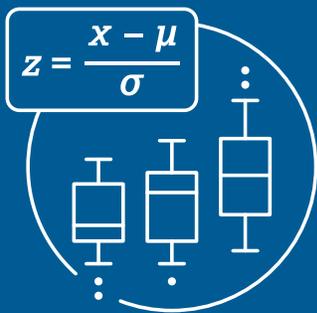
Reporting Category	Scoring Criteria	
<b>Part D (ii)</b> <b>(0-1 points)</b> <b>Point 8</b>	<b>0 points</b> Does not accurately <b>include</b> the context	<b>1 point</b> Accurately <b>includes</b> the context
	<b>Decision Rules and Scoring Notes</b>	
	<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response includes the incorrect context.</li> <li>▪ The response does not provide context.</li> <li>▪ The response does not include the population parameter.</li> </ul>	<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response includes refilling stations.</li> <li>▪ The response includes 2024 and 2025.</li> <li>▪ The response includes the population parameter.</li> </ul>
	<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "true mean difference between 2024 and 2025"</li> <li>▪ "difference in the number of refilling stations per office between 2024 and 2025"</li> <li>▪ "true mean difference in the number of refilling stations"</li> </ul>	<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "difference in the true mean number of refilling stations per office between 2024 and 2025"</li> </ul>

Reporting Category	Scoring Criteria	
<b>Part E</b> <b>(0-1 points)</b> <b>Point 9</b>	<b>0 points</b> Does not accurately <b>determine</b> whether a claim is supported or not supported based on statistical results	<b>1 point</b> Accurately <b>determines</b> whether a claim is supported or not supported based on statistical results
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately report that there is sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately reports that there is sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Erika cannot make any determination based on the data provided."</li> <li>▪ "There is not sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025."</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "There is sufficient statistical evidence of a difference in the true mean number of refilling stations per office in the country between 2024 and 2025."</li> <li>▪ "Erika has enough evidence to say that the true mean number of refilling stations per office in the country changed between 2024 and 2025."</li> </ul>
<b>Additional Notes:</b> <ul style="list-style-type: none"> <li>▪ Part E Points 9 and 10 can be earned in any order based on the response.</li> <li>▪ If the incorrect inference procedure was conducted in Part D or if the correct inference procedure provides incorrect results, use the students' results from the inference procedure to evaluate part E.</li> </ul>		

Reporting Category	Scoring Criteria	
<b>Part E</b> <b>(0-1 points)</b> <b>Point 10</b>	<b>0 points</b> Does not accurately <b>explain</b> the reasoning	<b>1 point</b> Accurately <b>explains</b> the reasoning
<b>Decision Rules and Scoring Notes</b>		
<b>Responses that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response does not accurately explain the reasoning to support the determination.</li> <li>▪ The response does not provide reasoning to support the determination.</li> </ul>		<b>Responses that earn this point:</b> <ul style="list-style-type: none"> <li>▪ The response accurately bases the explanation on whether 0 is in the interval and is consistent with part D.</li> </ul>
<b>Examples that do not earn this point:</b> <ul style="list-style-type: none"> <li>▪ "There is not enough data to make a decision."</li> <li>▪ "No, because 0 is not in the interval of <math>(-0.365, -0.015)</math>."</li> </ul>		<b>Examples that earn this point:</b> <ul style="list-style-type: none"> <li>▪ "Yes, because 0 is not in the interval of <math>(-0.365, -0.015)</math>."</li> </ul>

AP STATISTICS

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# Appendix

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**AP STATISTICS**

# **Formula Sheet and Tables**

# Formulas for AP Statistics

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## I. Descriptive Statistics

$$\bar{x} = \frac{1}{n} \sum x_i = \frac{\sum x_i}{n}$$

$$s = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2} = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\hat{y} = a + bx$$

## II. Probability and Distributions

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability Distribution	Mean	Standard Deviation
Discrete random variable, $X$	$\mu_X = E(X) = \sum x_i \cdot P(x_i)$	$\sigma_X = \sqrt{\sum (x_i - \mu_X)^2 \cdot P(x_i)}$
If $X$ has a binomial distribution with parameters $n$ and $p$ , then:		
$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ where $x = 0, 1, 2, 3, \dots, n$	$\mu_X = np$	$\sigma_X = \sqrt{np(1-p)}$

## III. Sampling Distributions and Inferential Statistics

$$\text{Standardized test statistic: } \frac{\text{statistic} - \text{parameter}}{\text{standard error of the statistic}}$$

$$\text{Confidence interval: } \text{statistic} \pm (\text{critical value})(\text{standard error of the statistic})$$

$$\text{Chi-square statistic: } \chi^2 = \sum \frac{(\text{Observed Count} - \text{Expected Count})^2}{\text{Expected Count}}$$

### III. Sampling Distributions and Inferential Statistics (cont.)

#### Sampling Distributions for Proportions

Sample Statistic	Mean	Standard Deviation	Standard Error
For one population: $\hat{p}$	$\mu_{\hat{p}} = p$	$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$	$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
For two populations: $\hat{p}_1 - \hat{p}_2$	$\mu_{\hat{p}_1 - \hat{p}_2} = p_1 - p_2$	$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$	$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$ When $p_1 = p_2$ is assumed: $SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\hat{p}_c(1-\hat{p}_c)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ , where $\hat{p}_c = \frac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$

#### Sampling Distributions for Means

Sample Statistic	Mean	Standard Deviation	Standard Error
For one population: $\bar{x}$	$\mu_{\bar{x}} = \mu$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$	$SE_{\bar{x}} = \frac{s}{\sqrt{n}}$
For two populations: $\bar{x}_1 - \bar{x}_2$	$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2$	$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$	$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$

# Tables for AP Statistics

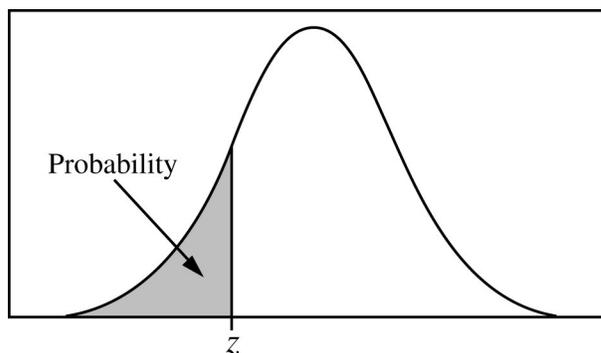


Table entry for  $z$  is the probability lying below  $z$ .

**Table A: Standard Normal Probabilities**

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

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**Table A: Standard Normal Probabilities (cont.)**

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

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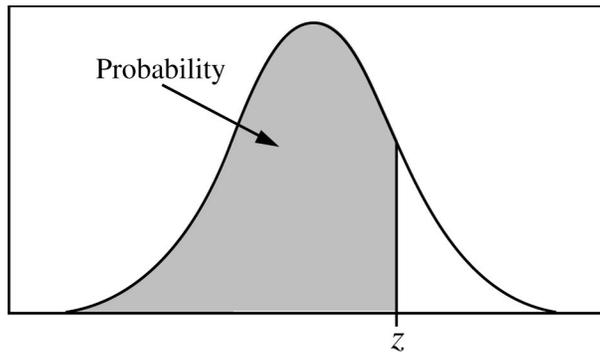


Table entry for  $Z$  is the probability lying below  $Z$ .

**Table A: Standard Normal Probabilities (cont.)**

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936

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**Table A: Standard Normal Probabilities (cont.)**

<b><i>z</i></b>	<b>.00</b>	<b>.01</b>	<b>.02</b>	<b>.03</b>	<b>.04</b>	<b>.05</b>	<b>.06</b>	<b>.07</b>	<b>.08</b>	<b>.09</b>
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

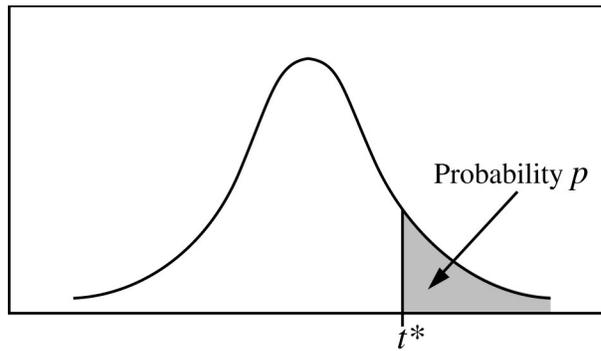


Table entry for  $p$  and  $C$  is the point  $t^*$  with probability  $p$  lying above it and probability  $C$  lying between  $-t^*$  and  $t^*$ .

**Table B:  $t$ -Distribution Critical Values**

$df$	Tail Probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850

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**Table B: *t*-Distribution Critical Values (cont.)**

<i>df</i>	Tail Probability <i>p</i>											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence Level <i>C</i>												

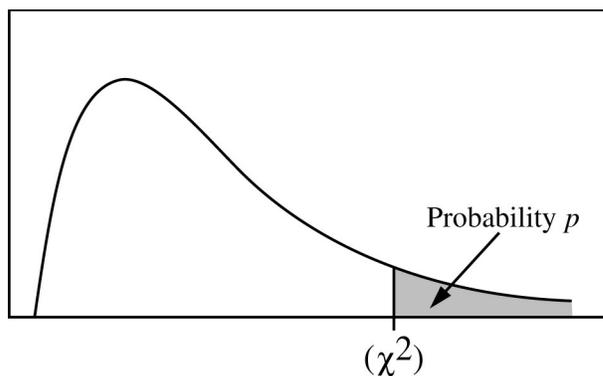


Table entry for  $p$  is the point  $(\chi^2)$  with probability  $p$  lying above it.

**Table C:  $\chi^2$  Critical Values**

$df$	Tail probability $p$											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50

*continued on next page*

**Table C:  $\chi^2$  Critical Values (cont.)**

<i>df</i>	Tail probability <i>p</i>											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

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