PROFESSIONAL DEVELOPMENT

AP® Physics 1

Rotational Motion

CURRICULUM MODULE

For the redesigned course launching fall 2014

The College Board
New York, NY

Revised spring 2015

About the College Board

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The College Board strongly encourages educators to make equitable access a guiding principle for their AP programs by giving all willing and academically prepared students the opportunity to participate in AP. We encourage the elimination of barriers that restrict access to AP for students from ethnic, racial and socioeconomic groups that have been traditionally underserved. Schools should make every effort to ensure their AP classes reflect the diversity of their student population. The College Board also believes that all students should have access to academically challenging course work before they enroll in AP classes, which can prepare them for AP success. It is only through a commitment to equitable preparation and access that true equity and excellence can be achieved.

Contents

Preface

AP® curriculum modules are exemplary instructional units composed of one or more lessons, all of which are focused on a particular curricular topic; each lesson is composed of one or more instructional activities. Topics for curriculum modules are identified because they address one or both of the following needs:

- A weaker area of student performance as evidenced by AP Exam subscores
- Curricular topics that present specific instructional or learning challenges

The components in a curriculum module should embody and describe or illustrate the plan/teach/assess/reflect/adjust paradigm:

1. *Plan* the lesson based on educational standards or objectives and considering typical student misconceptions about the topic or deficits in prior knowledge.
2. *Teach* the lesson, which requires active teacher and student engagement in the instructional activities.
3. *Assess* the lesson, using a method of formative assessment.
4. *Reflect* on the effect of the lesson on the desired student knowledge, skills, or abilities.
5. *Adjust* the lesson as necessary to better address the desired student knowledge, skills, or abilities.

Curriculum modules will provide AP teachers with the following tools to effectively engage students in the selected topic:

- Enrichment of content knowledge regarding the topic
- Pedagogical content knowledge that corresponds to the topic
- Identification of prerequisite knowledge or skills for the topic
- Explicit connections to AP learning objectives (found in the AP curriculum framework or the course description)
- Cohesive example lessons, including instructional activities, student worksheets or handouts, and/or formative assessments
- Guidance to address student misconceptions about the topic
- Examples of student work and reflections on their performance

The lessons in each module are intended to serve as instructional models, providing a framework that AP teachers can then apply to their own instructional planning.

Note on Internet Resources

All links to online resources were verified at the time of publication. In cases where links are no longer working, we suggest that you try to find the resource by doing a key-word Internet search.

— The College Board

Introduction

The AP Physics 1: Algebra-based curriculum includes learning objectives in rotational motion, a topic not previously part of the AP Physics B curriculum. The curriculum framework for the course — which can be found in the *AP Physics 1 and 2 Course and Exam Description* — addresses torque, angular motion (angular velocity, angular acceleration, and angular displacement), rotational inertia, rotational kinetic energy, and angular momentum (without vector algebra and vector calculus). Teachers who have not been teaching rotational motion will benefit from this curriculum module, as it includes suggestions for instructional practice, inquiry-based investigations, and formative and summative assessment materials to ease the transition to the new material in the curriculum framework. To limit the scope of this module, we focus only on rotational inertia, rotational kinetic energy, and conservation of angular momentum. This set of lessons should follow lessons on torque, forces and torques in equilibrium, and the basics of rotational kinematics (including use of θ, ω, and α in kinematic equations).

This curriculum module includes three lessons:

- Lesson 1: Rotational Inertia
- Lesson 2: Rotational Kinetic Energy
- Lesson 3: Changes in Angular Momentum and Conservation of Angular Momentum

Each lesson references the Physics 1 learning objectives that apply to that lesson. Other lesson components include the following:

- Demonstrations and class activities that can be conducted using inexpensive materials.
- Laboratory investigations, with the idea that you should provide the opportunity for students to design as much of each lab as possible themselves. The labs are identified and described in detail in the Appendices for your benefit. Depending on the experience of students, you should limit the amount of prompting and instead encourage students to design their experiments and/or plan data collection and analysis strategies. Students are also required to keep a permanent record of their laboratory experiments — either a written or digital portfolio or bound laboratory journal.
- Handouts and other resources for use as homework assignments or classroom activities.
- A formative assessment for each lesson. Formative assessments are low-stakes assessments that can help you identify areas of misunderstanding to address prior to — or during — the presentation of the lessons included here and determine students' readiness for the next lesson. Formative assessments provide valuable feedback to inform your instructional strategies based on student performance. They also give students insight into their learning progression and level of mastery of content and skills.

- Recommended websites that can help reinforce concepts when used in the classroom or as assigned work for students.

Connections to the Curriculum Framework

The selected learning objectives identified in each lesson are related to rotational motion. We assume that some of these objectives will have been addressed prior to the module, while others are being introduced for the first time in the module.

Appendix A

Each learning objective in the curriculum framework is linked with one or more science practices that capture important aspects of the work that scientists engage in. For a list of the AP Science Practices, see Appendix A or the curriculum framework in the *AP Physics 1 and 2 Course and Exam Description*. The science practices enable students to establish lines of evidence and use them to develop and refine testable explanations and predictions of natural phenomena. For example, Learning Objective 3.F.3.1, which involves rotational collisions, is linked with Science Practices 6.4 and 7.2. These science practices involve making claims and predictions about natural phenomena and connecting concepts across domains. Instruction about the concept of rotational collisions, then, should involve these science practices. Lesson 3 of this module, which covers rotational collisions and Learning Objective 3.F.3.1, illustrates this instructional approach; the recommended lab connects conservation of angular momentum to conservation of linear momentum. Assessment questions based on this learning objective might include application of the concept to an everyday situation, such as analyzing what happens on a child's merry-go-round ride when a person steps onto the ride while it is rotating. In designing classroom examples and illustrations and structuring formative assessments, you should consider both the learning objectives and the science practices.

The following boundary statement from the curriculum framework helps to define the extent of the use of vector notation in AP Physics 1:

Quantities such as angular acceleration, velocity, and momentum are defined as vector quantities, but in this course the determination of "direction" is limited to clockwise and counterclockwise with respect to a given axis of rotation.

Instructional Time and Strategies

This unit, with the activities and related laboratory work, will require approximately 2 weeks, with 10 standard 50-minute classroom periods. The times given for the demonstrations, activities, and labs are estimates for student work during class time and do not include construction or setup time by the teacher.

Though some teachers may wish to begin the year in Physics 1 with an integrated study of the kinematics of linear and rotational motion, it is highly recommended that several units precede this unit on rotational motion — particularly if AP Physics 1 is the first course in physics for students. The development of conceptual and mathematical skills through the study of linear motion provides essential background for the study of rotational motion. Students first need to develop understanding of kinematics, Newton's laws, work and energy, and conservation of linear momentum in order to effectively draw the analogies between linear and rotational motion described in this module. It is also important for students to have developed skills in laboratory design and analysis prior to this unit. It is assumed that students have prior knowledge of rotational kinematics and the symbols and applications of kinematic equations to rotational motion. Laboratory experiments in which students have investigated torques and static equilibrium (such as construction of mobiles and related calculations) would provide good preparation for the progression to rotational motion, accelerated systems, and conservation laws applied to rotational motion.

Equation Tables

Equations that students might use in solving problems or answering questions will be provided for students to use during all parts of the AP Physics 1 Exam. It is not intended for students to memorize the equations, so teachers can feel comfortable in allowing students to use the AP Physics 1 equation tables for all activities and assessments. For the AP Physics 1 equation tables, see Appendix B or the *AP Physics 1 and 2 Course and Exam Description*.

Appendix B

Lesson 1: Rotational Inertia

Guiding Questions

- How is rotational inertia related to the inertia discussed in the context of Newton's first law of motion?
- How is the rotational inertia of an object or system related to the structure of that object or system?
- How does the rotational inertia of an object or system affect the object's or system's motion?

Lesson Summary

This lesson sets the stage for the study of rotational kinetic energy and angular momentum by developing student understanding of the rotational inertia of an object or system. Students should have previously learned that the mass of an object describes the object's inertia when in translational motion. In this lesson, students learn that rotational inertia describes, in a similar manner, the rotational motion of an object. For example, an object with larger rotational inertia may be more difficult (require more torque) to start moving, but once it starts moving it requires a larger torque (such as frictional torque) to stop. Friction forces play a role in the motion of an object in translational motion in the same way that frictional torques affect rotation of an object or system. For example, when a wheel is rotating, friction forces exert torques on the wheel to cause it to eventually stop. It is particularly important in this unit for you to continually reinforce the similarities between concepts related to translational motion — with which students should be familiar — and those related to rotational motion, which is new and rather unfamiliar to students.

Connections to the Curriculum Framework

Learning objectives related to the topic of rotational inertia covered in this lesson are identified below:

- **Learning Objective (4.D.1.2):** The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. [See Science Practices 3.2, 4.1, 4.2, 5.1, and 5.3]

This learning objective has two parts: (a) describing a model of a rotational system, and (b) using that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. In this lesson, the focus is on the description of a rotational system (i.e., using the rotational inertia of a system as a description of the internal structure of the system). The second part of the learning objective will be applied in Lesson 3, where rotational inertia is used as part of the analysis of changes in angular momentum due to interactions of objects or systems with rotational inertia.

- **Learning Objective (5.E.2.1):** The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. [See Science Practice 2.2]

This learning objective also applies to two lessons in this module. The description of rotational inertia is addressed in this lesson, and the description of angular momentum is addressed in Lesson 3. This learning objective also clarifies the scope of the types of systems that should be included. For example, in accordance with the learning objective, the student might be told that a compound object, such as a hammer, has a certain rotational inertia. The student then might be asked to reason about the rotational inertia of the hammer, when rotated around the end of its handle, if the handle of the hammer is shortened. (Answer: The rotational inertia would be lowered, since the value of r in the equation for the rotational inertia of the form $I = kmr^2$ would be less.) Or the student might be given the scenario of a wheel with the axis of rotation through the center and asked how the rotational inertia of the wheel would be different if the rubber rim was replaced with a lead metal rim. (Answer: The rotational inertia would be greater — this time due to an increase in m in the equation.)

Student Learning Outcomes

As a result of this lesson, students should be able to:

- Explain how mass, radius, and internal structure can be used to describe the rotational inertia of an object or system
- Relate how rotational inertia affects the motion of an object or system

Student Prerequisite Knowledge

Prior to this lesson, students should have learned and understood basic concepts about objects in translational motion as well as the following concepts specifically related to torque and rotational motion, as addressed in the curriculum framework:

- Fundamentals of torque and equilibrium
- Distinctions between translational and rotational equilibrium
- Kinematics with rotational quantities and their symbols (τ, α, ω, θ)
- Changes in rotational acceleration and rotational velocity due to torques exerted on the object or system
- Conversions between linear and angular quantities
- Experimental design and data collection/analysis related to torques (e.g., construction of mobiles)

You can use Handout 1, "Using the Symbols of Rotational Motion," as a formative assessment to check which of these ideas should be reviewed with students prior to this lesson.

Handout 1

Common Student Misconceptions and Challenges

The most common misconceptions related to this lesson involve the failure to realize that rotational inertia is a property of an object. Students are familiar with inertia, which depends on mass. Rotational inertia depends not only on mass but also on the distribution of mass around the axis of rotation of the object. For this reason, the activities in Lesson 1 need to emphasize that objects with the same mass (the petri dishes, the rotating batons, and the rotating eggs) can behave differently.

Furthermore, it's important to show students that changing the axis of rotation (such as grasping one of the batons by the end instead of by the middle) will change the baton's rotational inertia. Students often mistakenly answer that an object does not exhibit its rotational inertia unless it is rotating, so it is important for you to emphasize that this property with respect to a given axis of rotation is the same whether the object is stationary or rotating.

Materials and Resources Needed

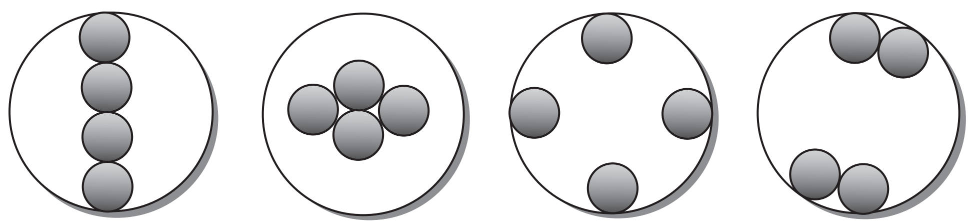
- Petri dishes with lids or similar closed, flat cylinders (four or multiples of four)
- Steel balls (four per petri dish or cylinder, with the diameter of each ball equal to the height of the container)
- Balance
- Two pieces of $\frac{1}{2}$″ or $\frac{3}{4}$″ schedule 40 PVC pipe (each cut to a length of 1.5 m)
- Four end caps to fit the PVC pipes
- Four "concrete anchors" (can be found in hardware stores; should fit firmly inside the PVC pipes without sliding)
- Duct tape
- Several eggs, some raw and some hard-boiled
- Board, 1″ $\times$ 6″, about 2 m long
- Commercially available solid disk-and-hoop set
- Varied objects to determine rotational inertia (e.g., basketball, baseball, wooden dowels of varied diameters and lengths)
- Handouts 1 and 2 and Appendix C (Note: Materials for handouts and appendices are listed separately on those documents.)

Activity 1: Mass Distribution and Rotational Inertia, Part 1

[Time: 45 min]

This activity gives students an informal "feel" for how mass distribution affects rotational inertia. Prior to the activity, prepare one or more sets of four petri dishes (or cylindrical containers with similar dimensions such as hand cream containers) by first painting the interiors of the petri dishes so they are opaque. Inside each dish, glue in identical steel balls — four in each dish, placed in different orientations. For example, one dish might have the four steel balls spaced equidistant around the inside perimeter. Another might have the four steel balls placed in a line across the diameter. (See suggested arrangements in Figure 1.) Seal the dishes so students cannot see the contents.

Figure 1: Suggested setup for mass distribution and rotational inertia, part 1.



After the students have assured themselves that the dishes all have the same mass, assign them the task of working in teams to devise experimental methods to qualitatively determine the internal structures of the dishes. Students will observe that the dishes behave differently when they roll the dishes on their sides. Each dish exhibits a different behavior depending upon its rotational inertia. For a large class, make several identical sets of dishes so students can compare results, with the ultimate goal of matching all of them with their possible configurations.

Activity 2: Qualitative Lab — Introduction to Rotational Inertia

Handout 2

[Time: 40 min]

The series of investigations in Handout 2 ("Qualitative Lab — Introduction to Rotational Inertia"), with questions for the student, is designed to reinforce the concept of rotational inertia. Most of the investigations require students to make an initial prediction, to describe and explain their observations of the experiment, and to reconcile their predictions with their observations.

▶ *Formative Assessment*

Students can write their responses to the questions in the lab and submit them to you for review, or small-group representatives can share responses with the whole class as the basis for class discussion. Students would benefit from writing their responses in paragraph form in a journal or report for you to evaluate, as this is an important aspect of testing in Physics 1. In either scenario, you should identify misconceptions and bring these to the attention of the individual student or to the class. If you identify misconceptions shared by a large number of students, additional explanation of the concept followed by repetition of the activity provides students the opportunity to retest and reevaluate their conclusions.

Handout 3 ("Formative Assessment on Torque and Rotational Kinematics") also contains questions that may be used for formative assessment.

Handout 3

Activity 3: Demonstration — Mass Distribution and Rotational Inertia, Part 2

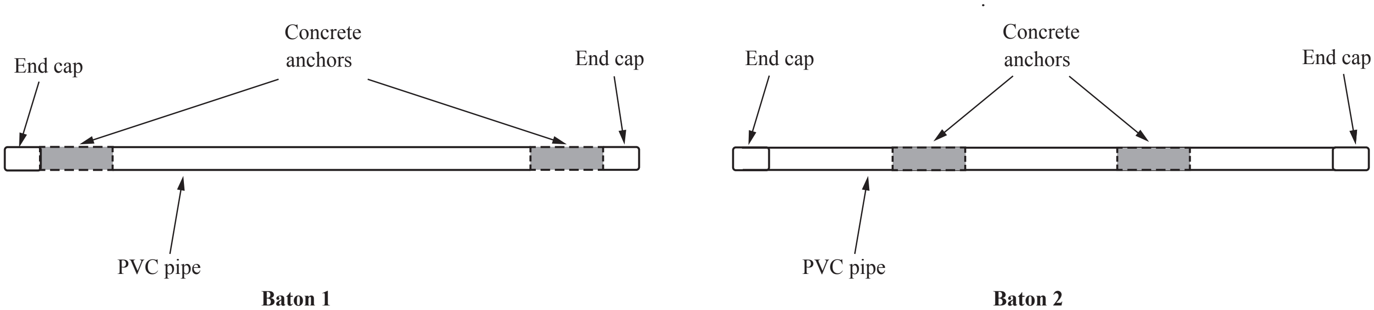
[Time: 5 min]

This short demonstration uses two batons that have the same mass and are made of the same materials but have different rotational properties, as a result of how the batons are constructed. (This demonstration is similar to an activity in Handout 2, in which students use two metersticks, one with identical masses attached to the ends and one with identical masses attached closer to the middle. However, this demonstration is more interesting to students in that the internal structure is not immediately apparent. If students have completed Handout 2, this demonstration will enable you to see how well students understood the main points of the activity in the handout.)

Handout 2

To prepare the first baton, place a concrete anchor (or another object that will barely fit inside the PVC pipe) just inside each end of one pipe. (See Figure 2.) The anchors should fit in the pipe securely without sliding. Close the pipe with end caps and secure the caps with tape for safety. Repeat the process for the second baton, but use a dowel to push the anchors closer toward the middle of the pipe.

Figure 2: Batons for mass distribution and rotational inertia, part 2.



Have two students stand at the front of the class, and give each one a baton. Instruct the students to grasp the baton at the center and rotate the baton first 180 degrees in one direction, then 180 degrees in the other direction. Ask the students to have a race to see who can get the baton rotating back and forth the fastest; then have the students switch batons and describe to the class what they have experienced. The baton with the anchors near the ends should have a noticeably higher rotational inertia and be more difficult for the student to start rotating or to switch direction.

Activity 4: Demonstration — The Rotating Eggs

[Time: 5 min]

Bring in two hard-boiled eggs and two raw eggs. (Make sure there are no identifying features, such as printed labels, that students can use to tell the eggs apart.) Tell students it is their task to determine which of the four are hard-boiled without breaking them. Students may offer other suggestions, such as trying to float the eggs. Assure students that this method is not reliable, as eggs often take in water during boiling. On a level surface, give each egg a twist to start it spinning, and once it is spinning, lightly touch it on the top to stop it. Ask students to analyze and explain what they have observed. Students should conclude that the raw eggs are noticeably more difficult to start or stop. In the raw egg, gradually the liquid insides begin to spin as they are dragged around by the shell. Once the raw egg starts spinning, the denser portions of the interior move as far from the axis of rotation as they can get, increasing the rotational inertia of the egg — making it greater than the fixed rotational inertia of the hard-boiled egg. As a result, the raw egg is harder to start and stop spinning.

Activity 5: The Ring and Disk Race

[Time: 20 min]

This activity requires a circular metal ring and solid cylindrical disk of the same mass and diameter, which are available from most science supply companies. (These can also be constructed, making sure the outer edge of each has a coating that provides the same coefficient of friction so that students do not consider that as another variable.) Set up a long ramp (or use a smooth, sloped section of floor in a hallway). Set the ring and the disk vertically beside each other at the top of the ramp and release them simultaneously. Have students observe which starts rolling more quickly (i.e., the one with less rotational inertia) and the one that starts rolling more slowly but which ultimately rolls longer before frictional torque can stop it (i.e., the one with more rotational inertia). Then ask students to explain what they observed. If the ring and disk are nearly identical in size and mass, the gravitational force exerted at the center of mass of each provides equal torques on the two objects. The students can assume, then, that if the angular accelerations are different, the differences in distribution of mass cause the objects to have different rotational inertia.

A similar activity — but one that has a more complex explanation — is the popular soup can race. For more information, see Stanley Micklavzina's article "It's in the Can: A Study of Moment of Inertia and Viscosity of Fluids" (details provided in the References section).

Activity 6: Determining Rotational Inertia of Standard Shapes

[Time: 20 min]

Once students have gained some practice with the concept of rotational inertia, they can start taking measurements and calculating the rotational inertia for each of several common objects with regular shapes, such as a basketball with the axis through its center ($I = \frac{2}{3}mr^2$), a baseball with the axis through its center ($I = \frac{2}{5}mr^2$), a bicycle wheel with thin spokes ($I = mr^2$), a wooden rod with the axis through its center of mass and perpendicular to its length ($I = \frac{1}{12}mr^2$), and a wooden rod with the axis at one end ($I = \frac{1}{3}mr^2$). Provide students with the general formula for each of the shapes. (Note: Students will not be required to memorize values of rotational inertia for various shapes or to derive rotational inertia using the parallel axis theorem.)

Activity 7: Lab — Determination of Rotational Inertia

[Time: 50 min]

Appendix C

Students are now ready to design and conduct an experiment (see Appendix C, "Lab — Determination of Rotational Inertia") to determine the rotational inertia of an object. The following directive can be given to students: "Using the equipment available, design an experiment to determine the rotational inertia of the object." In this experiment, the shape of the object studied is irregular, which challenges students to determine the value of I experimentally rather than calculating it. The calculations are somewhat involved, with equations new to students, so you might be a little more directive than usual in providing the necessary equations as a prompt to students. If students are provided with precut PVC pieces and all materials, 50 minutes should be adequate to gather data. However, extra time will be required if you wish to have students cut PVC pieces and assemble materials, if necessary — perhaps another 30 minutes. Students will also need additional time to complete the analysis in the laboratory journal, usually outside of class.

▶ *Formative Assessment*

Laboratory work can be used as a type of formative assessment where students apply their knowledge to the design of a related investigation. Any of the following formative assessment methods might be used to evaluate students' readiness to proceed to the next lesson:

- You can use the prompts in the rotational inertia lab in Appendix C to derive a set of follow-up questions to determine students' mastery of concepts and their readiness to apply the concept of rotational inertia in

Appendix D

Lesson 2. Questions can be similar to those asked in the above activities, with students writing their answers individually and you leading a whole-class discussion of the answers. If many students perform poorly on questions covering a given concept, you should provide further explanation of the concept; then, assign another activity on that concept and review students' answers before proceeding to Lesson 2. One possibility is the activity in Appendix D, "Class Activity — Determining the Angular Speed of a Fan." This activity takes about 15 minutes to conduct.

- Because this is a full lab, it is expected that students will write a lab report that includes an analysis (i.e., calculations, observations, conclusions). You should evaluate these reports and respond individually to students — either directly on that report or on a rubric — providing feedback to each student on correct use of terms and correct calculations, in addition to giving an analytical summary that reveals any misconceptions the student might harbor. Misconceptions that will compromise the student's ability to progress in the subsequent lesson should lead the student back to the lab for further observation or back to the report to rethink conclusions.
- Post-lab peer reports, with a representative from each lab group presenting the group's conclusions either orally or on a whiteboard, provide both you and the students with the opportunity to give feedback to individual groups on their experimental design and conclusions. The goal is for the peer-review process to be constructive, allowing groups to rethink their conclusions if necessary, make amendments where appropriate, and hear about alternate methods of solution. This post-lab activity can require an additional 20 to 30 minutes, depending on the size of the class and number of groups reporting.

Lesson 2: Rotational Kinetic Energy

Guiding Questions

- How is total energy of a rotating object calculated?
- How does torque do work to change the energy of a rotating system?

Lesson Summary

In this lesson, students learn to describe and calculate the total energy of a rolling object, using both translational and rotational kinetic energies (e.g., for a ball rolling across a floor). They also apply the work–energy theorem to examine how torque can do work in changing the rotational kinetic energy of an object or system.

Connections to the Curriculum Framework

In this lesson, students will use the following learning objectives to gain an understanding of rotational kinetic energy and of how this knowledge allows for a more complete explanation of the total energy of an object or system.

- **Learning Objective (5.B.4.2):** The student is able to calculate changes in kinetic energy and potential energy of a system, using information from representations of that system. [See Science Practices 1.4 and 2.1]
- **Learning Objective (5.B.5.4):** The student is able to make claims about the interaction between a system and its environment in which the environment exerts a force on the system, thus doing work on the system and changing the energy of the system (kinetic energy plus potential energy). [See Science Practices 6.4 and 7.2]

Student Learning Outcomes

As a result of this lesson, students should be able to:

- Explain how external forces exerted on a system can exert torques to change the state of rotation of the system
- Design an experiment to explore the effect of external torques on rotational kinetic energy

- Calculate changes in energy of a system (rotational kinetic energy, translational kinetic energy, potential energy) when provided with angular or linear velocity and a formula to calculate rotational inertia of the system

Student Prerequisite Knowledge

This lesson assumes that students have the following prerequisite skills and knowledge from the previous lesson or from earlier work in the course:

Appendix B

- Familiarity with concepts and equations for rotational kinematics, particularly rotational velocity (ω) and rotational inertia (I), in order to calculate rotational kinetic energy, $K = \frac{1}{2} I\omega^2$. (See the AP Physics 1 equation tables in Appendix B.)
- Ability to relate linear velocity (v) to angular velocity (ω) for a point on the rotating object, using $v = \omega r$, to make conversions between linear and angular quantities. (This does not include center-of-mass speed of an object in rotational and translational motion.)
- Experience with the work–energy theorem as it relates to linear motion, in order to form the analogous relationship for angular motion.

Handout 1

You can use Handout 1, "Using the Symbols of Rotational Motion," as an individual assignment to assess whether students have this prerequisite knowledge. If not, the same handout can be used as a small-group activity to bring all students up to speed.

Common Student Misconceptions and Challenges

The most common misconception in this lesson is that an object with rotational motion cannot also have translational motion, or vice versa. Another misconception is to ignore or misunderstand the role of friction in generating rotational motion. Consider asking: "What happens if a ball is placed on a ramp and there is no friction between the ball and the ramp?" The student must realize that without friction, the ball will slide down the ramp rather than rolling. The friction force provides torque to cause the ball to roll rather than slide.

Materials and Resources Needed

- Basketball
- Appendix E and Handout 3 (Note: Materials for handouts and appendices are listed separately on those documents.)

Activity 1: Demonstration — Rotational Kinetic Energy

[Time: 10 min]

A rotating object or system has rotational kinetic energy that is calculated in a method analogous to translational kinetic energy. In translational motion, kinetic energy depends on translational inertia or mass (m) and linear speed (v): $K = \frac{1}{2} mv^2$. In rotational motion, kinetic energy depends on rotational inertia (I) and rotational speed (ω): $K = \frac{1}{2} I\omega^2$. Of course, an object can be moving in linear (or translational) motion and angular (or rotational) motion at the same time, so the total kinetic energy is the sum.

To emphasize the analogy, roll a basketball across the floor and ask students to describe the energy of the basketball — and what happens to that energy as the ball rolls to a stop. The gravitational potential energy does not change, and the total kinetic energy is the sum of rotational and translational kinetic energies ($K_{total} = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$). The mechanical energy is converted to thermal energy of the molecules of the ball and floor as the ball stops (due to negative work done by the friction force).

Activity 2: Lab — Using Rotational Kinetic Energy for the Ball on a Ramp Lab

[Time: 80 min]

Appendix E

Students can revisit labs they may have completed previously in the course in which they have used rolling objects — and disregarded rotational motion — by designing a lab that includes both the linear and angular motions of a rolling object. Appendix E, "Lab — Using Rotational Kinetic Energy for the Ball on a Ramp Lab," describes an experiment in which students let a ball roll down a ramp and off a tabletop, measuring where the ball lands, to figure out what fraction of the system's initial potential energy converts to rotational kinetic energy instead of translational kinetic energy. Students can design the experiment using available materials and making decisions about what measurements to take, which calculations are pertinent, and how many trials are advisable.

Formative Assessment

As discussed in the previous lesson, laboratory work can be used as a type of formative assessment where students apply their knowledge to the design of a related investigation. Any of these follow-up activities might be used to evaluate students' readiness to proceed to the next lesson:

- You can use the prompts and questions found in Appendix E to derive a set of questions to determine students' mastery of concepts and their readiness to apply the concept of rotational kinetic energy. If many students perform poorly on these questions, you should provide further explanation of the concept; then, assign another activity on that concept and review students' answers before proceeding to the next lesson.

- Because this is a full lab, it is expected that students will write a lab report that includes an analysis (i.e., calculations, observations, conclusions). You should evaluate these reports and respond individually to students — either directly on that report or on a rubric — providing feedback to each student on correct use of terms and correct calculations, in addition to giving an analytical summary that reveals any misconceptions the student might harbor. Misconceptions that will compromise the student's ability to progress in the next lesson should lead the student back to the lab for further observation or back to the report to rethink conclusions.
- Post-lab peer reports, with a representative from each lab group presenting the group's conclusions either orally or on a whiteboard, provide both you and the students with the opportunity to give feedback to individual groups on their experimental design and conclusions. The goal is for the peer-review process to be constructive, allowing groups to rethink their conclusions if necessary, make amendments where appropriate, and hear about alternate methods of solution. This post-lab activity can require an additional 20 to 30 minutes, depending on the size of the class and number of groups reporting.

Lesson 3: Changes in Angular Momentum and Conservation of Angular Momentum

Guiding Questions

- How are rotational collisions analogous to collisions of objects or systems in linear motion?
- How does torque change angular momentum?
- How do objects or systems interact to change angular momentum?

Lesson Summary

In this lesson, the concepts of linear momentum and conservation of linear momentum, with which students should be familiar, are shown to be analogous to the angular quantities — angular momentum and conservation of angular momentum.

Connections to the Curriculum Framework

This lesson covers several learning objectives, with strong emphases on predictions, descriptions, and experimental design. Writing, in paragraph form or in a laboratory report (or journal), is also emphasized in this lesson. The learning objectives covered in Lesson 3 are as follows:

- **Learning Objective (3.F.3.1):** The student is able to predict the behavior of rotational collision situations by the same processes that are used to analyze linear collision situations using an analogy between impulse and change of linear momentum and angular impulse and change of angular momentum. [See Science Practices 6.4 and 7.2]
- **Learning Objective (3.F.3.2):** In an unfamiliar context or using representations beyond equations, the student is able to justify the selection of a mathematical routine to solve for the change in angular momentum of an object caused by torques exerted on the object. [See Science Practice 2.1]

- **Learning Objective (3.F.3.3):** The student is able to plan data collection and analysis strategies designed to test the relationship between torques exerted on an object and the change in angular momentum of that object. [See Science Practices 4.1, 4.2, 5.1, and 5.3]
- **Learning Objective (4.D.1.1):** The student is able to describe a representation and use it to analyze a situation in which several forces exerted on a rotating system of rigidly connected objects change the angular velocity and angular momentum of the system. [See Science Practices 1.2 and 1.4]
- **Learning Objective (4.D.1.2):** The student is able to plan data collection strategies designed to establish that torque, angular velocity, angular acceleration, and angular momentum can be predicted accurately when the variables are treated as being clockwise or counterclockwise with respect to a well-defined axis of rotation, and refine the research question based on the examination of data. [See Science Practices 3.2, 4.1, 4.2, 5.1, and 5.3]
- **Learning Objective (4.D.2.1):** The student is able to describe a model of a rotational system and use that model to analyze a situation in which angular momentum changes due to interaction with other objects or systems. [See Science Practices 1.2 and 1.4]
- **Learning Objective (4.D.2.2):** The student is able to plan a data collection and analysis strategy to determine the change in angular momentum of a system and relate it to interactions with other objects and systems. [See Science Practice 4.2]
- **Learning Objective (4.D.3.1):** The student is able to use appropriate mathematical routines to calculate values for initial or final angular momentum, or change in angular momentum of a system, or average torque or time during which the torque is exerted in analyzing a situation involving torque and angular momentum. [See Science Practice 2.2]
- **Learning Objective (4.D.3.2):** The student is able to plan a data collection strategy designed to test the relationship between the change in angular momentum of a system and the product of the average torque applied to the system and the time interval during which the torque is exerted. [See Science Practices 4.1 and 4.2]
- **Learning Objective (5.E.1.1):** The student is able to make qualitative predictions about the angular momentum of a system for a situation in which there is no net external torque. [See Science Practices 6.4 and 7.2]
- **Learning Objective (5.E.1.2):** The student is able to make calculations of quantities related to the angular momentum of a system when the net external torque on the system is zero. [See Science Practices 2.1 and 2.2]
- **Learning Objective (5.E.2.1):** The student is able to describe or calculate the angular momentum and rotational inertia of a system in terms of the locations and velocities of objects that make up the system. Students are expected to do qualitative reasoning with compound objects. Students are expected to do calculations with a fixed set of extended objects and point masses. [See Science Practice 2.2]

LESSON 3: Changes in Angular Momentum and Conservation of Angular Momentum

Student Learning Outcomes

As a result of this lesson, students should be able to:

- Calculate net torque on a system and use torque to calculate change in angular momentum
- Plan experiments or data collection strategies or analyze data for changes in angular momentum due to torque exerted on the system
- Apply the concept of conservation of angular momentum to interactions of objects

Student Prerequisite Knowledge

In readiness for this lesson, the student should be able to:

- Use kinematic equations for angular motion
- Describe or calculate rotational inertia for an object or extended system
- Design experiments and analyze data related to rotational kinematics
- Make analogies between linear and angular motion
- Use torques to determine changes in rotational motion

Handout 1

You can use Handout 1, "Using the Symbols of Rotational Motion," as an individual assignment to assess whether students have this prerequisite knowledge. If not, the same handout can be used as a small-group activity to bring all students up to speed. Lessons 1 and 2 above should provide students with all the prerequisite knowledge needed; from students' written lab reports and from class discussions, the teacher can identify any of the prerequisite ideas that may need to be reviewed.

Common Student Misconceptions and Challenges

Handout 1

In this lesson, students may have difficulty picturing changes in angular momentum as clockwise or counterclockwise. (Note that the directions of rotation pseudovectors, as determined by the right-hand rules, are outside the scope of this course.) The symbols for rotational quantities may also produce challenges, as more equations and relationships are added. Using or reviewing Handout 1 would be helpful, as this lesson completes the unit.

Materials and Resources Needed

- Low-friction rotating stool or platform
- Hand weights
- Bicycle wheel with a handle installed on each end of the axle
- Handouts 4 and 5

Activity 1: Demonstration — The Rotating Stool

[Time: 15 min]

Have a student sit on a low-friction rotating stool with a hand weight in each hand. (For safety, use the type of exercising hand weight with a handle on which the student can get a good grip.) Have the student position his or her center of mass over the stool, and assign other students to stand nearby to spot the student to prevent him or her from falling. Ask the student to sit squarely with arms and legs extended. Start the student's rotation, and once the student is rotating, ask the student to pull his or her limbs in close to the body in a tucking motion. (Prepare the student by explaining why he or she will need to lean forward to re-center when the arms and legs are pulled in — a good review lesson on center of mass.) The class should observe an increase in rotational speed when the rotating student pulls in his or her limbs and a decrease in speed when the student extends them. Ask students to relate this to conservation of angular momentum. They should come to the understanding that because $L = I\omega$, a decrease in radius (with limbs pulled in) also decreases rotational inertia of the stool-student-weights system. As rotational inertia decreases, angular speed increases — and vice versa.

Following the demonstration, encourage students to relate the demonstration to various other examples. For instance, a rotating diver or acrobat pulls in his or her limbs to decrease rotational inertia ($I = kmr^2$) by decreasing radius, thus increasing angular speed by conservation of angular momentum ($I_0\omega_0 = I_f\omega_f$). During his or her time in the air, the athlete can then make more turns.

Also guide students to relate conservation of angular momentum to the Earth making its orbit around the Sun. When the Earth is closest to the Sun (at perihelion, around January 2 of each year), it is moving fastest in its orbit; when the Earth is at its farthest point from the Sun (at aphelion, around July 3 of each year), it is moving slowest in its orbit. Again, by conservation of angular momentum, when the radius is smallest the rotational inertia is smallest and angular velocity is largest. (Remind students that angular velocity is proportional to linear velocity: $v = \omega R$.)

During this activity, take time to discuss with students the common threads between linear momentum and angular momentum. Point out that in the same manner that external forces exerted on an object or system will change the linear momentum of a system, external torques exerted on a system will change the angular momentum of the system. So angular momentum does not remain constant in the presence of external torques.

Activity 2: Demonstration —The Rotating Stool and Torque

[Time: 10 min]

Have a student sit on a low-friction rotating stool, holding a low-friction bicycle wheel with a handle installed on each end of the axle. As in the previous activity, have the student position his or her center of mass over the stool, and assign other students to stand nearby to spot the student. Ask the student to sit squarely and extend the wheel outward with both hands. Start the wheel rotating and ask the student to quickly flip the wheel by 180 degrees. The student should start rotating on the chair. Then ask the student to flip the wheel back quickly. This should generate a large rotational impulse in the opposite direction, causing the student to reverse the direction of rotation. To explain this, apply Newton's second law to rotational motion (see the AP Physics 1 equation tables in Appendix B).

$$\text{linear:} \quad F = \frac{\Delta p}{\Delta t}$$

$$\text{rotational:} \quad \tau = \frac{\Delta L}{\Delta t}$$

Students should recognize that the change in angular momentum of the wheel — in a short amount of time — exerts a torque on the student, causing the student to rotate. A faster switch in direction of the spinning wheel should cause a more noticeable change in rotation of the student. Assuming negligible friction in the bearings of the rotating stool, there is no external torque on the wheel-student-stool system, so angular momentum is conserved.

▶ *Formative Assessment*

This formative assessment — which brings in elements of all three lessons — consists of one free-response question (see Handout 4, "Formative Assessment on Rotational Inertia, Kinematics, Kinetic Energy, and Momentum"). The question is selected from a previous AP Physics C Exam and modified to fit the standards for AP Physics 1. Use this question prior to the summative assessment on these lessons to determine whether students can correctly relate linear force to torque and also apply the concepts of rotational inertia, torque related to work done in increasing rotational kinetic energy, and how torque exerted on a system can change the angular momentum of that system. If students do not perform well when using specific concepts, you might review the analogous linear equations and concepts and the relevant row from Handout 1, "Using the Symbols of Rotational Motion." After giving this assessment, it is important to work through each part carefully with students to review concepts and equations prior to the summative assessment.

Appendix B

Handout 4

Handout 1

Summative Assessment

[Time: 1 hour]

Handout 5

Handout 5 serves as the summative assessment for this module. The first two questions have been selected from previous AP Physics C Exams and modified to fit the standards for AP Physics 1, and the third question is written in the style of the laboratory question that will be part of each AP Physics 1 Exam. The provided answers emphasize the concepts included in the Physics 1 learning objectives and are therefore the main idea you should look for in grading students' answers.

References

AP Physics 1 and 2 Course and Exam Description. New York: The College Board, 2014. The *AP Physics 1 and 2 Course and Exam Description* contains the curriculum framework and science practices, describes the course, and includes sample exam questions.

Micklavzina, Stanley J. "It's in the Can: A Study of Moment of Inertia and Viscosity of Fluids." *Physics Education* 39, no. 1 (2004): 38–39. This article explains why the "standard" explanation (in terms of rotational inertia) of which soup can wins the race is oversimplified and sometimes wrong.

Resources

"Angular Momentum" and "Torques and Gyroscopes." From *The Mechanical Universe … and Beyond*. California Institute of Technology and Intelecom, 1985. Accessed December 18, 2013. http://www.learner.org/resources/series42.html. The *Mechanical Universe* series consists of half-hour videos that illustrate physics concepts.

Ehrlich, Robert. *Turning the World Inside Out and 174 Other Simple Physics Demonstrations*. Princeton, NJ: Princeton University Press, 1990. This has been a source of inspiration for several of the demonstrations in this module.

"Ladybug Revolution." PhET. University of Colorado at Boulder. Accessed December 18, 2013. http://phet.colorado.edu/en/simulation/rotation. The PhET website contains interactive simulations, such as this one, that can be used during class or assigned outside of class for students. Teacher resources and student handouts are also available.

"Rotational Translation." HippoCampus. Accessed December 18, 2013. www.hippocampus.org/Physics. (The program is available under the headings "General Physics" and "Circular Motion and Rotation.") The Hippocampus website has short programs such as this one that summarize concepts and contain interactive assessment questions and problems. This program takes less than 5 minutes and explains the relationship between rotational and translational motion.

Handout 1

Using the Symbols of Rotational Motion

Complete the following table. Some of the cells are already filled in for you.

	Linear or Translational			Angular or Rotational		
Concept	Symbol	Formula	Unit	Symbol	Formula	Unit
Displacement	s	$s = v_0 t + \frac{1}{2}at^2$ $r = r_0 + v_0 t + \frac{1}{2}at^2$	m	θ	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	°, rad
Velocity				ω		rad/s
Acceleration				α		rad/s^2
Inertia	m		kg	I	$I = kmr^2$	kg·m^2
Force/torque (Newton's second law)	F	$\Sigma F = ma$	N	τ	$\tau = I\alpha$ $\tau = r_\perp F = rF\sin\theta$	N·m
Kinetic energy	K			K		
Work	W			W		
Momentum	P		N·s *or* kg·m/s	L		kg·m^2/s
Centripetal acceleration	a_c			a_c	$a_c = \omega^2 r$	
Power			W	P		

Helpful Equations and Notes

$$a = \alpha r$$

$$v = \omega r$$

$$s = \theta r$$

α, ω, and θ *must* be in radian measure

Rotational Inertia for Common Objects, I = *kmr*^{2}	
Solid sphere	$I = \frac{2}{5}mr^2$
Hollow sphere	$I = \frac{2}{3}mr^2$
Hoop (central axis) or solid object moving in a circle	$I = mr^2$
Hoop (rotating around its diameter)	$I = \frac{1}{2}mr^2$
Solid cylinder or disk (central axis)	$I = \frac{1}{2}mr^2$

Handout 1 Answer Key

	Linear or Translational			Angular or Rotational		
Concept	**Symbol**	**Formula**	**Unit**	**Symbol**	**Formula**	**Unit**
Displacement	s	$s = v_0 t + \frac{1}{2}at^2$ $r = r_0 + v_0 t + \frac{1}{2}at^2$	m	θ	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	°, rad
Velocity	v	$v = v_0 + at$	m/s	ω	$\omega = \omega_0 + \alpha t$	rad/s
Acceleration	a	$a = \frac{\Sigma F}{m}$		α	$\alpha = \frac{\Sigma\tau}{I}$	rad/s^2
Inertia	m		kg	I	$I = kmr^2$	kg·m^2
Force/torque (Newton's second law)	F	$\Sigma F = ma$	N	τ	$\tau = I\alpha$ $\tau = r_\perp F = rF\sin\theta$	N·m
Kinetic energy	K	$K = \frac{1}{2}mv^2$	J	K	$K = \frac{1}{2}I\omega^2$	J
Work	W	$W = \Delta E = F\Delta r$ $= Fr\cos\theta$	J	W	$W = \Delta E = \tau\Delta\theta$	J
Momentum	p	$p = mv$ $\Delta p = F\Delta t$	N·s *or* kg·m/s	L	$L = I\omega$ $\Delta L = \tau\Delta t$	kg·m^2/s
Centripetal acceleration	a_c	$a = \frac{v^2}{r}$	m/s^2	a_c	$a_c = \omega^2 r$	m/s^2
Power	P	$P = W/t = \Delta E/t$ $= Fv$	W	P	$P = W/t = \Delta E/t = \tau\omega$	W

Handout 2

Qualitative Lab — Introduction to Rotational Inertia

Objects resist changes in motion, and objects with more mass have more of this resistance. For instance, even on a low-friction surface, it's harder to get a brick moving than it is to get a small wood block moving. It's also harder to stop the brick once it's already moving than it is to stop the block. In other words, the brick has more *inertia* than the block; it's harder to change the motion of a brick.

The same idea applies to objects undergoing rotational motion instead of linear motion. It's harder to get a bowling ball spinning in place than it is to get a basketball spinning in place. It's also harder to stop the spinning bowling ball. In other words, the bowling ball has more resistance to changes in rotation motion — more *rotational inertia* — than a basketball.

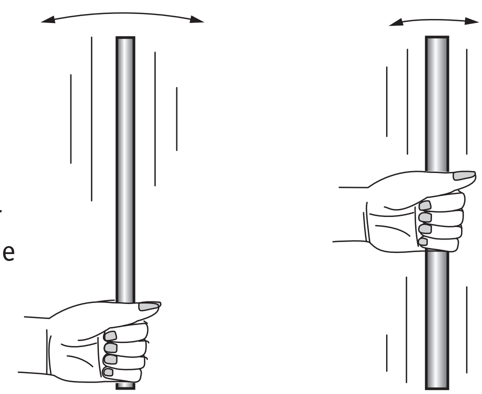
Directions: In this lab, you'll perform a series of investigations involving rotational inertia to deepen your conceptual understanding of this idea and to understand why the mathematical equations for rotational inertia make sense. For each investigation, read the steps of the investigation and the questions first before carrying out the procedure. In some cases, you will be asked to make a prediction before you perform the investigation. After each investigation, answer the questions about what you observed.

Materials: Two identical rods or metersticks, clamps or small weights and tape, broom

Part 1: Rotating Rods

A. You'll need a rod (or meterstick). To conduct the experiment, you will first hold the rod near one end and use your wrist to spin the rod back and forth in a horizontal plane, letting the rod swing through about 20 degrees before reversing direction. Then, you will repeat the process holding the rod near its middle. Your teacher will demonstrate these steps for you. To make sure gravity plays a minimal role, you can use the lab bench to help you support the rod as you move it.

If you let your wrist exert equal effort both times, then you apply the *same torque* both times.



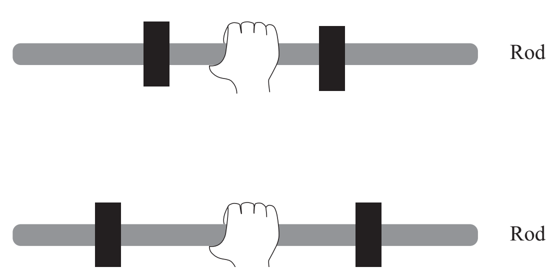
Top-down view

Make a prediction: In which case will the rod spin back and forth more easily? Why?

Do it: What did you observe during this investigation?

Follow-up discussion: Why was the rod easier to spin back and forth in one case compared with the other?

B. Use two identical rods (or metersticks) and attach two masses to each of them as shown. (For the masses, clamps or small weights attached with tape work well.) On each rod, the two masses should both be placed the same distance from the center. But on Rod 2, both



masses should be placed farther from the center than they are on Rod 1. You will hold each rod at its center and spin it back and forth through a small angle as in section A above.

Make a prediction: Which rod will be harder to spin? Why?

Do it: What did you observe during this investigation?

Follow-up discussion: Can you explain all the results you've seen so far?

C. A friend says: "Rotational inertia is just a fancy name for an object's mass when the object is rotating. If the object has more mass, it has more rotational inertia, and if it has less mass, it has less rotational inertia — end of story. Nothing else matters."

In what ways, if any, do you agree with your friend? In what ways, if any, do you disagree?

Part 2: Broom

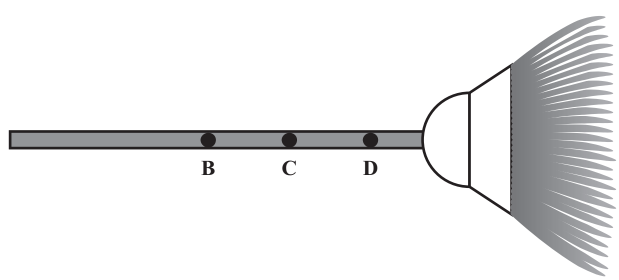
A. In this activity, you will spin a light (...spin a light broom...) broom back and forth in a horizontal plane, as you did with the rod in part 1 above. You will hold the broom first at the brush end and then at the handle end.

Make a prediction: In which case will the broom be easier to spin back and forth? Why?

Do it: What did you observe during this investigation?

Follow-up discussion: Explain what you observed, and relate your explanation to your explanations from part 1.

B. The diagram below shows a broom with points labeled B, C, and D. Point C is the same distance from both ends of the broom. In this activity, you'll hold a broom at one of these points and spin the broom horizontally as you've done previously. You can let an end of the broom slide across the lab bench or floor. This will make it less of an issue when the broom is "out of balance" in your hand.



Make a prediction: At which point should you hold the broom so that it will be easiest to spin it back and forth? Why?

Do it: After testing your prediction, describe what you observed. How can you be sure that your prediction about what would happen didn't influence your observations?

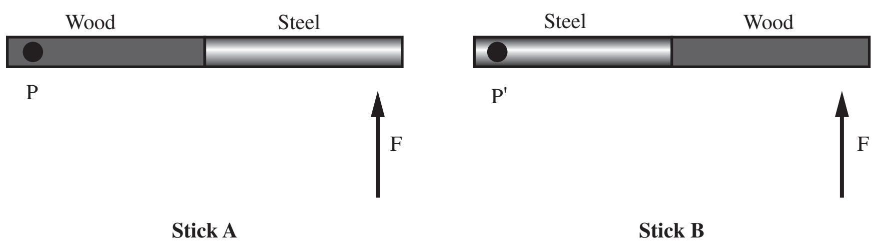
Follow-up discussion: Explain your observations in a way that fits in with your earlier explanations in this lab.

C. Hold the broom with the handle vertically aligned, and rotate it around (a) the vertical axis of rotation through the center of mass, and (b) a horizontal axis through the center of mass. Explain why it's easier to rotate the broom around one of these axes than the other, and make sure your explanation connects to those you've given so far in this lab.

D. Look back over part 2B above, and then hold the broom at each of the three points labeled in the figure. You can confirm that, of those three points, the point around which it was easiest to rotate the broom is also the point for which the broom is best "balanced" in your hand. Explain.

Part 3: Review

The figure below shows two identical sticks, both half wood and half steel, with Stick A pivoted at its wood end and Stick B pivoted at its steel end. Identical torques are exerted on the two sticks. Which stick, if either, undergoes a greater angular acceleration?



Handout 2 Answer Key

Part 1

A. The rod should be easier to spin when held near its middle because the rotational inertia of the rod is smaller with respect to an axis through its center.

B. Rod 2 should be harder to spin because more of its mass (taking the clamps into account) is farther from the axis of rotation, making its rotational inertia larger. As in section A, the object with larger rotational inertia — with more of its mass distributed farther from the axis of rotation — is harder to angularly accelerate.

C. The friend is right that, other things being equal, a more massive object has more rotational inertia. Sections A and B, however, demonstrate that two objects of the *same* mass can also have different rotational inertias. Rotational inertia is larger when a greater fraction of the object's mass is farther from the axis of rotation.

Part 2

A. The broom should be harder to spin when held by the handle end because, in that case, a greater percentage of the broom's mass — namely, the brush — is farther from the axis of rotation.

B. The broom should be easiest to rotate when held at point D. With the axis of rotation through D, a greater percentage of the broom's mass — most of which is contained in the brush — is closer to the axis of rotation.

C. The broom is easier to rotate around the vertical axis because, in that case, all its mass is within 15 or 20 cm of the axis of rotation.

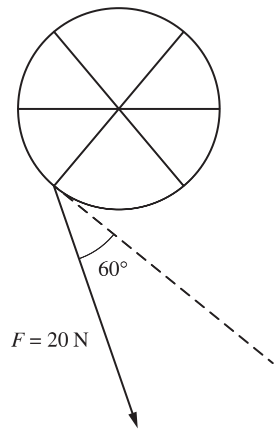
D. Roughly speaking, the balancing point, called the center of mass, is the point with respect to which the average distance of the bits of mass in the broom is minimized. Because the rotational inertia is smaller when more of the bits of mass are closer to the axis of rotation, it makes sense that the broom is easier to spin around point D than around B or C.

Part 3

Stick B undergoes a greater angular acceleration because it has a smaller rotational inertia — that is, it puts up less "resistance" to having its rotational motion changed. Stick B has a smaller rotational inertia because a greater percentage of its mass — the steel rather than the wood — is closer to the axis of rotation through the pivot.

Handout 3

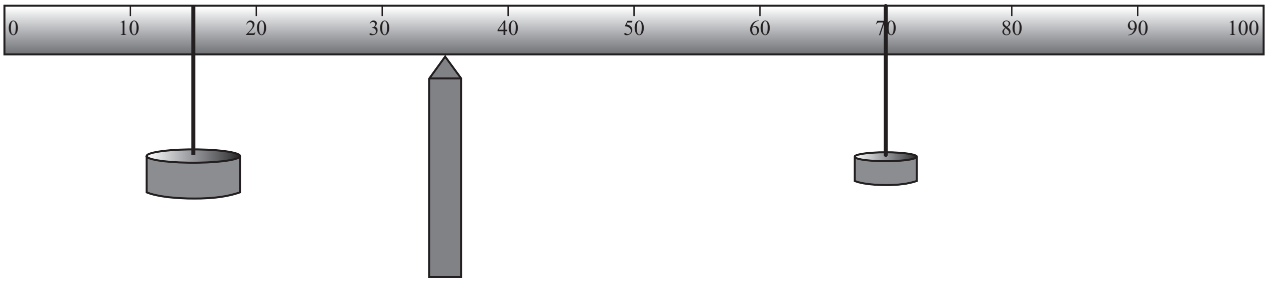
Formative Assessment on Torque and Rotational Kinematics



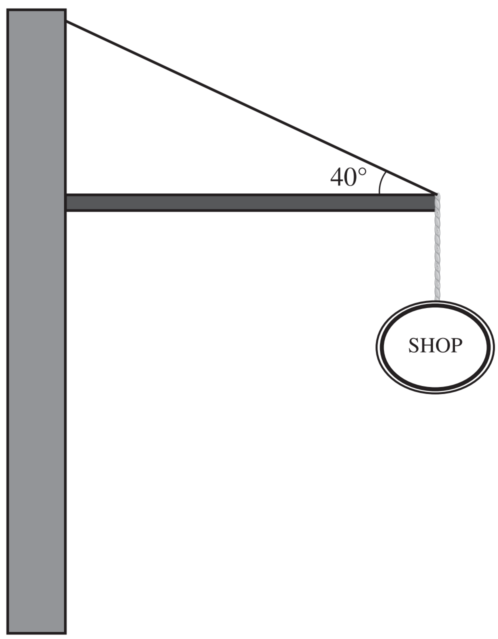
1. A string is attached to a nearly frictionless wheel, and a 20 N force is applied at a 60° angle to the tangent, as shown above. The diameter of the wheel is 1.0 meter. What is the torque exerted on the wheel by the string?
   (a) 5 N·m
   (b) 8.7 N·m
   (c) 10 N·m
   (d) 20 N·m
   (e) 40 N·m

2. A basketball with a mass of 0.60 kg and radius of 7 cm is rolling across a level floor at a constant speed of 2.0 m/s.
   (a) Determine the ball's angular velocity.
   (b) What is the ball's angular acceleration?
   (c) How many turns will the ball make in 2 seconds?

3. A bicycle wheel with a radius of 0.5 meter and mass of 3.0 kg is turning at 20 rpm when the rider applies the brakes. The wheel turns 10 more times before the bicycle comes to a stop.
   (a) What is the wheel's angular acceleration?
   (b) How far has the wheel traveled across the surface?



4. The uniform meterstick above has an object with mass 800 grams hanging at the 15 cm mark and an object with mass 350 grams at the 70 cm mark. It balances horizontally on a pivot placed at the 35 cm mark. What is the mass of the meterstick?



5. A uniform wooden beam with a mass of 20 kg extends horizontally from a wall, as shown above. A support cable (of negligible mass) extends from the far end of the beam to the wall, forming a 40° angle with the beam. The beam has a sign with a mass of 5 kg hanging from the end of it.
   (a) Find the tension in the cable that helps to support the beam and sign.
   (b) Find the horizontal and vertical components of the force the wall exerts on the wooden beam.

Handout 3 Answer Key

1. Answer choice (a). $\tau = R_{\perp}F = (0.5\text{ m})(20\text{ N})(\cos 60°) = 5\text{ N}\cdot\text{m}$

2. (a) $\omega = \frac{v}{R} = \frac{2\text{ m/s}}{0.07\text{ m}} = 28.6\text{ rad/s}$

   (b) zero (The ball is moving at constant speed.)

   (c) $\theta = \omega t = (28.6\text{ rad/s})(2.0\text{ s}) = 57.2\text{ rad}$

   $(57.2\text{ rad})\left(\frac{1\text{ rotation}}{2\pi\text{ rad}}\right) = 9.12\text{ rev}$

3. First, find the initial angular speed in radians per second:

   $\left(\frac{20\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{2\pi}{3}\text{rad/s}$

   Then substitute into the equations and solve for angular acceleration and linear displacement:

   (a) $\omega_f^2 = \omega_0^2 + 2\alpha\theta$

   $0 = \left(\frac{2\pi}{3}\text{rad/s}\right)^2 + 2(\alpha)(20\pi\text{ rad})$

   $\alpha = -\frac{\pi}{90}\text{rad/s}^2$

   (b) $s = R\theta$

   $s = (0.5\text{ m})(20\pi\text{ rad}) = 10\pi\text{ m} = 31\text{ m}$

4. $\tau_{\text{clockwise}} = \tau_{\text{counterclockwise}}$

   $(m_{350})(g)(0.35\text{ m}) + (m_{\text{stick}})(g)(0.15\text{ m}) = (m_{800})(g)(0.2\text{ m})$

   $m_{\text{stick}} = 816\text{ g}$

5. (a) Let L equal the length of the beam:

   $\tau_{\text{clockwise}} = \tau_{\text{counterclockwise}}$

   $(m_{\text{sign}})(g)(L) + (m_{\text{beam}})(g)(\frac{1}{2}L) = (T\sin 40°)(L)$

   (Note: Use the component of the tension perpendicular to the beam at the point of attachment. The length of the beam is not needed because L cancels in every term.)

   $T = 229\text{ N}$

   (b) The horizontal and vertical components of forces exerted on the beam must balance.

   $F_{\text{wall}}x = T(\cos 40°) = 229\text{ N} \times (\cos 40°) =$ **<u>175 N</u>**

   $F_{\text{wall}}y + T(\sin 40°) = (20\text{ kg}) \times 9.8 + (5\text{ kg}) \times 9.8$

   $F_{\text{wall}}y + T(\sin 40°) = 245\text{ N}$

   $F_{\text{wall}}y = 245\text{ N} - (229\text{ N} \times \sin 40°)$

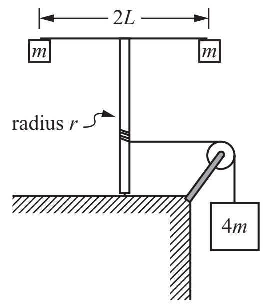
   $F_{\text{wall}}y = 245\text{ N} - 147\text{ N}$

   $F_{\text{wall}}y =$ **<u>98 N</u>**

Handout 4

Formative Assessment on Rotational Inertia, Kinematics, Kinetic Energy, and Momentum

Free-Response Question



A light string that is attached to a large block of mass $4m$ passes over a pulley with negligible rotational inertia and is wrapped around a vertical pole of radius r, as shown in Experiment A above. The system is released from rest, and as the block descends the string unwinds and the vertical pole with its attached apparatus rotates. The apparatus consists of a horizontal rod of length $2L$, with a small block of mass m attached at each end. The rotational inertia of the apparatus is assumed to be $2mL^2$.

(a) If the downward acceleration of the large block is measured to be a, determine the tension T in the string, in terms of the acceleration of the falling block.
(b) Determine the torque exerted on the rotating pole by the string, in terms of the mass of the blocks and the acceleration.
(c) When the large block has descended a distance D, how does the instantaneous rotational kinetic energy of the apparatus compare with the value $4mgD$? Check the appropriate space below and justify your answer.

____Greater than $4mgD$ ____Equal to $4mgD$ ____Less than $4mgD$

Now consider the experiment again, this time including the rotational inertia of the small pulley.

(d) If the rotational inertia of the pulley were large enough to have an effect on the experiment, would you predict your answers to be different? Explain your response.
(e) How will the angular velocity of the rotating apparatus and linear velocity of the falling mass compare now with their values calculated in parts (a), (b), and (c)?
(f) Discuss how the torque exerted by the string on the rotating apparatus changes the angular momentum of the apparatus.

Handout 4 Answer Key

(a) First express Newton's law for the motion of the large block as it falls:

$\Sigma F = ma$

Then substitute the values for the forces on the block, making the downward motion positive:

$4mg - T = 4ma$

$T = 4mg - 4ma$

(b) Calculate the torque the tension force exerts on the rotating pole:

$\tau = Tr = (4mg - 4ma)r$

(c) The decrease in gravitational potential energy of the large block is equal to the increase in translational kinetic energy of the large block plus the rotational kinetic energy of the apparatus:

$\Delta U_G = \Delta K_{\text{rotation}} + \Delta K_{\text{translation}}$

$4mgD = K_{\text{rotation}} + \frac{1}{2}(4m)v^2$

Therefore, the rotational kinetic energy of the apparatus is less than $4mgD$, because part of the gravitational potential energy is in the translational kinetic energy of the large block as it falls.

(d) All answers would be different. There are three parts to this answer. Now that part of the change in gravitational potential energy is used to do work in rotating the pulley, the change in rotational kinetic energy of the rotating apparatus is less, so the linear and angular acceleration are less. In part (a), the tension would not be the same in both ends of the string; there would be greater tension on the falling block than on the rotating apparatus because there would need to be a net tension force on the small pulley to cause it to rotate. In part (b), the *m, g*, and *r* would be the same, but acceleration *a* would be less, because the falling block now has to accelerate both the rotating apparatus and the rotating pulley. In part (c), assuming we still allow the large mass to fall a distance *D*, the change in gravitational potential energy would be the same, but some of the kinetic energy would now be in the rotating pulley.

(e) Because the acceleration is less, as described in part (d), if the same falling distance *D* is used, then the final velocity is also less:

$v_f^2 = v_0^2 + 2aD$

If the final linear velocity of the falling mass is less, then the final angular velocity of the rotating device is also less:

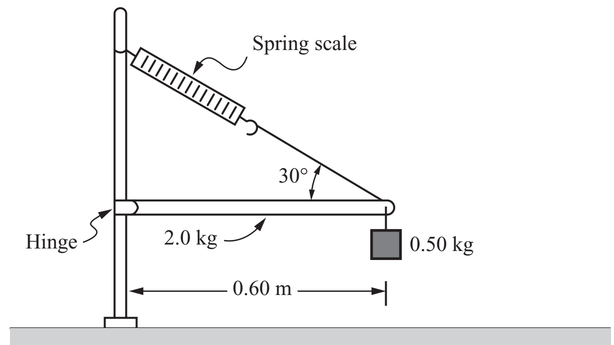
$v = \omega r$

(f) Just as constant net force exerted on a system changes its linear momentum at a steady rate ($\Delta p = F\Delta t$), the constant torque exerted by the string on the apparatus increases its angular momentum at a steady rate:

$\Delta L = \tau\Delta t$

Handout 5

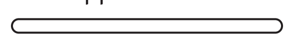
Summative Assessment



1. Free-Response Question

The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.

(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force as a vector originating at its point of application.



(b) Calculate the reading on the spring scale, which represents the tension in the cord.

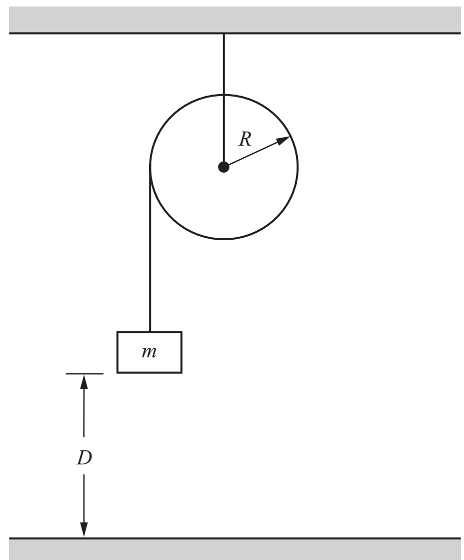
(c) The cord that supports the rod is cut near the end of the rod and the block is removed from the rod before the rod is released and allowed to rotate about the hinge. Discuss whether the initial (linear) acceleration of the center of mass of the rod about the hinge is equal to g, greater than g, or less than g. Explain your answer. (The rotational inertia of the rod with the axis at the hinge is $\frac{1}{3}mL^2$.)

(d) Consider again the hinged rod (without cord or block attached).

i. When the rod is released from a horizontal position, is the angular acceleration of the end of the rod greater than, less than, or equal to the angular acceleration of the center of mass of the rod? Explain your answer.

ii. When the rod is released from a horizontal position, is the linear acceleration of the end of the rod greater than, less than, or equal to the linear acceleration of the center of mass of the rod? Explain your answer.

2. Free-Response Question

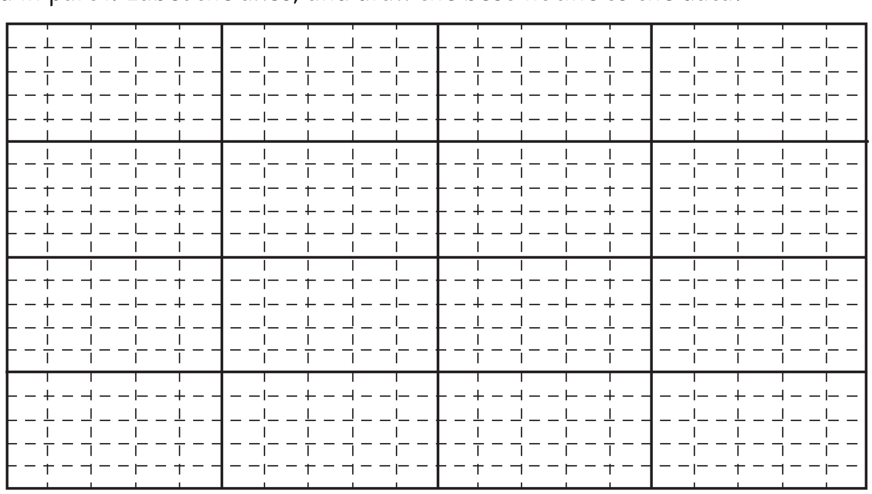


A solid disk of unknown mass and known radius R is used as a pulley in a lab experiment, as shown above. A small block of mass m is attached to a string, the other end of which is attached to the pulley and wrapped around it several times. The block of mass m is released from rest and takes a time t to fall the distance D to the floor. Provide all answers in terms of the given quantities (R, m, t, D) and fundamental constants.

(a) Calculate the linear acceleration a of the falling block in terms of the given quantities.
(b) The time t is measured for various heights D and the data are recorded in the following table.

D (m)	t (s)
0.5	0.68
1.0	1.02
1.5	1.19
2.0	1.41

i. How could the variables used in part (a) be graphed in order to determine the acceleration of the block, using a best-fit line?
ii. On the grid below, plot the measured quantities for the variables listed in the table in the manner described in part i. Label the axes, and draw the best-fit line to the data.



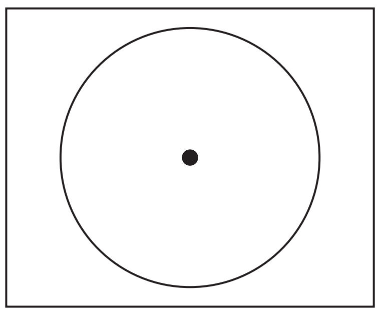
iii. Use your graph to calculate the magnitude of the acceleration.

(c) The value of acceleration found in part (b)iii, along with numerical values for the given quantities, is used to determine the rotational inertia of the pulley:

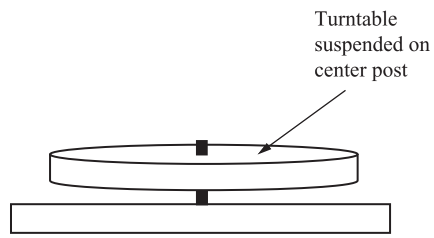
$$I = mR^2\left(\frac{g}{a} - 1\right)$$

The pulley is removed from its support and its rotational inertia is found to be greater than the value calculated from the experimental data. Give one explanation for this discrepancy.

3. Free-Response Laboratory Question



Top view



Side view

A very light low-friction cylindrical turntable is mounted on a sturdy base. Although the turntable is well balanced, its density might not be uniform. Design an experimental method to gather data that can be used to determine the rotational inertia of the turntable. In each part, provide the explanations and/or diagrams necessary to support your response.

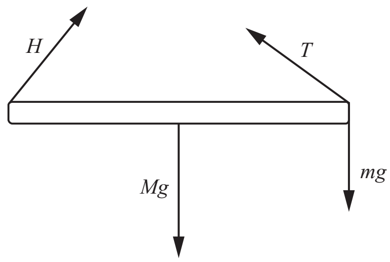
(a) First, list all equipment you will use and describe what measurements you will take and how you will take them, in enough detail so that another student could carry out your experiment. Use diagrams to clarify your experimental setup.

(b) Explain how you will use your measurements to determine the rotational inertia. Again, provide enough details that another student could carry out the calculations and/or produce the representations (e.g., graphs) that you discuss.

(c) Now you will consider the uncertainty in the rotational inertia.
  i. Which measurement might contribute most to that uncertainty, and why?
  ii. In doing or redoing the experiment, how could you reduce the uncertainty stemming from the measurement you selected in part i?

Handout 5 Answer Key

1. (a)



(b) Show that all net torques and forces must equal zero for equilibrium.

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma \tau = 0$

Set the pivot at the left end of the rod and solve, using the torque equation.

$Mg(\frac{1}{2}L) + mg(L) = T(\sin 30°)L$

$\frac{1}{2}Mg + mg = T(\sin 30°)$

$\frac{1}{2}(2\text{ kg})(9.8\text{ m/s}^2) + (0.5\text{ kg})(9.8\text{ m/s}^2) = T(\sin 30°)$

$T = 29\text{ N}$

(c) With the attached cord and block removed, the only torque to accelerate the rod is the weight of the rod. Since $\tau = I\alpha$, calculate angular acceleration first.

$\alpha = \frac{\tau}{I} = \frac{Mg(\frac{1}{2}L)}{\frac{1}{3}ML^2} = \frac{3g}{2L}$

Since the answer requires linear acceleration,

$a = \alpha R = \alpha(\frac{1}{2}L)$

$a = (\frac{1}{2}L)(3g)/2L = \frac{3}{4}g$

The acceleration is less than g.

(d) i. The angular acceleration of every point on the rod is the same, since the rod moves as one object sweeping the same angle during each time interval.

ii. The linear acceleration of a point on the rod depends on the distance of that point from the hinge ($a = \alpha r$). Because the end of the bar is twice as far from the hinge and the center of mass of the rod, the linear acceleration of the end of the rod is twice the linear acceleration of the center of mass.

2. (a) Start with the general equation for the motion and then substitute D for Δy and 0 for v_{0y}.

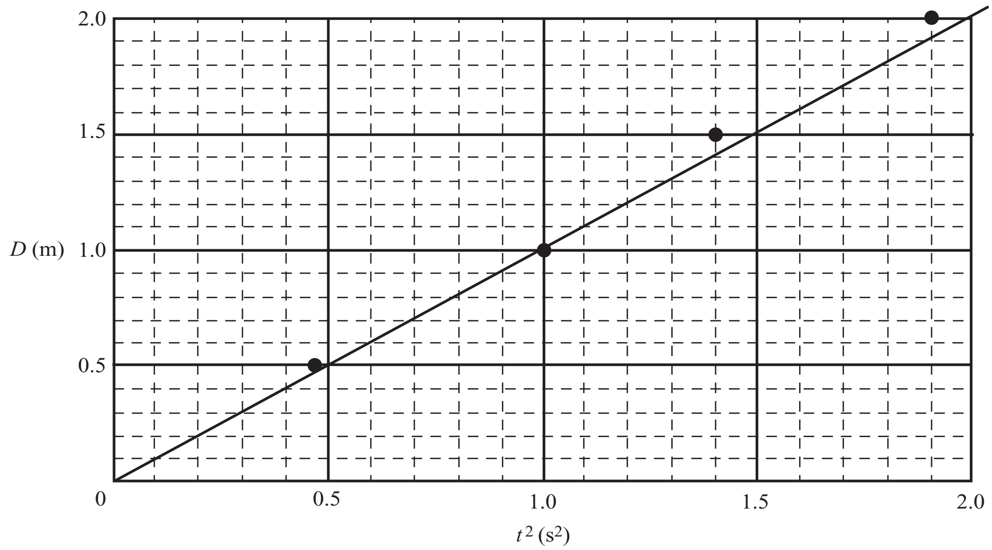
$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$

$D = \frac{1}{2}a_y t^2$

$a_y = \frac{2D}{t^2}$

(b) i. Examine the equation $D = \frac{1}{2}a_y t^2$ to see that it is in linear form if D is considered to be a function of t^2. So graphing "D versus t^2" would produce a line.

ii. The student should use the data table provided and plot the given values for D on the y-axis and plot values for t^2 on the x-axis. The student will then need to construct a best-fit line through those data points either in this section or in the next section to take the slope. (A sample graph is shown here.) If the student plots "t^2 as a function of D," which is also a correct method, the slope will be equal to $2/a$.



Data points on the graph are given below:

D (m)	t^2 (s^2)
0.5	0.46
1.0	1.04
1.5	1.42
2.0	1.99

iii. Since the equation is of the form $D = \frac{1}{2}at^2$, the graph of "*D* as a function of t^2" has a slope equal to $\frac{1}{2}a$.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{2.0 - 0.5}{1.8 - 0.5} = 1.2\,\text{m/s}^2$$

$$a = 2.4\,\text{m/s}^2$$

(Note: It is also acceptable for the student to graph $\sqrt{D}$ as a function of *t* to get a best-fit line, in which case the slope is equal to $\sqrt{\frac{1}{2}a}$, which will yield a correct value for *a,* but the relationship is more difficult for students to see.)

(c) Examine the equation for rotational inertia to determine what experimental results in measurements of *m*, *R*, or *a* would cause the value of *I* to be calculated lower than the actual value:

$$I = mR^2\left(\frac{g}{a} - 1\right)$$

(Note that students would not be responsible for knowing or deriving this equation.) It is important to first realize that the fraction *g/a* is greater than 1 because the falling block has tension on it and thus accelerates at a value less than *g*. Values of *m* or *R* that are too small or a value of *a* that is too large would produce calculations of rotational inertia from experimental data that are smaller than the actual value. There are several possible responses, including (1) wrapping string around the pulley makes the effective value of *R* larger than the actual value, so torque was larger than calculated, making inertia seem smaller; and (2) slipping of the string on the pulley causes the acceleration to be greater. (Note: Friction at the pulley is not a correct answer, since friction would cause acceleration to be smaller.)

3. (a) The response to part (a) should include the following:
    - A reasonable method to exert a set of torques on the turntable and make measurements that can be used to calculate angular momentum. (E.g., Attach a hanging mass at the end of a string hanging over the edge of the table, with the string wrapped around either the center post or outside edge of the turntable, and allow the mass to fall from rest. Then time a certain distance of fall.)
    - A method to measure torques that can lead to reasonably accurate values. (Note: Using a spring scale at the end of a string wrapped around either the center post or the perimeter might produce less reasonable values than suspending hanging masses over the edge of the base and determining acceleration of each mass, but it is an acceptable answer.)
    - Multiple trials (at least three) with different values for hanging mass (*not* just measuring the same thing multiple times) or varied methods of exerting torques from which values are averaged.
    - A list of all equipment needed and diagrams appropriate to the method described above. Some students may decide to use labeled diagrams as a way of listing equipment. (E.g., In a diagram of the top view of the turntable, the student shows the string wrapped around either the center post or the turntable perimeter. The student labels the radius R. In a diagram of the side view, the student shows a mass hanging from the string, indicating the distance D, and diagrams or mentions the stopwatch needed to measure time.)

   (b) The response to part (b) should include the following:
    - All necessary equations that would be needed to answer the question (depending on method and measurements described above), without extraneous equations.
    - A method to calculate a value for rotational inertia from torque and angular acceleration data.
    - A method to calculate change in angular momentum from previous data.
    - Description of a graph that correctly represents the function (i.e., the dependence of change in angular momentum on torque exerted on the turntable, so the student should define torque as the independent variable to be graphed on the y-axis).

A sample response to part (b) is shown below:

- $D = \frac{1}{2}at^2$, where D is distance mass falls and t is time required. Calculate the acceleration a. From it, calculate the disk's angular acceleration using $a = \alpha R$.
- $Mg - T = Ma$, where M is falling mass, T is string tension, and a is acceleration of falling mass. Calculate the tension T. From that tension, calculate the torque on the disk using $\tau = RT$, where R is radius.
- So, for each mass M used in the experiment, we obtain a torque τ and an angular acceleration α. Plot τ as a function of α. Draw the best-fit line, which should pass through the origin. Because $\tau = I\alpha$, the slope of the best-fit line gives the rotational inertia.

   (c) i. The student selects an appropriate measurement based on the experimental design described. The student then accurately explains how a measurement that is too high or too low would contribute to the uncertainty in the calculation of rotational inertia (e.g., if the time for fall is measured low, the acceleration a will be too large, leading to a larger change in angular momentum).

   ii. At least one step is described that would make *a* change in the uncertainty discussed in part i (e.g., taking many more trials for time and averaging them to reduce the effect of a single measurement error, using an electronic photogate, or making sure the timing measurements are taken in such a way to avoid parallax).

Appendix A

Science Practices for AP Courses

Science Practice 1: The student can use representations and models to communicate scientific phenomena and solve scientific problems.

- 1.1: The student can *create representations and models* of natural or man-made phenomena and systems in the domain.
- 1.2: The student can *describe representations and models* of natural or man-made phenomena and systems in the domain.
- 1.3: The student can *refine representations and models* of natural or man-made phenomena and systems in the domain.
- 1.4: The student can *use representations and models* to analyze situations or solve problems qualitatively and quantitatively.
- 1.5: The student can *reexpress key elements* of natural phenomena across multiple representations in the domain.

Science Practice 2: The student can use mathematics appropriately.

- 2.1: The student can *justify the selection of a mathematical routine* to solve problems.
- 2.2: The student can *apply mathematical routines* to quantities that describe natural phenomena.
- 2.3: The student can *estimate numerically* quantities that describe natural phenomena.

Science Practice 3: The student can engage in scientific questioning to extend thinking or to guide investigations within the context of the AP course.

- 3.1: The student can *pose scientific questions*.
- 3.2: The student can *refine scientific questions*.
- 3.3: The student can *evaluate scientific questions*.

Science Practice 4: The student can plan and implement data collection strategies appropriate to a particular scientific question.

- 4.1: The student can *justify the selection of the kind of data* needed to answer a particular scientific question.
- 4.2: The student can *design a plan* for collecting data to answer a particular scientific question.
- 4.3: The student can *collect data* to answer a particular scientific question.
- 4.4: The student can *evaluate sources of data* to answer a particular scientific question.

Science Practice 5: The student can perform data analysis and evaluation of evidence.

- 5.1: The student can *analyze data* to identify patterns or relationships.
- 5.2: The student can *refine observations and measurements* based on data analysis.
- 5.3: The student can *evaluate the evidence provided by data sets* in relation to a particular scientific question.

Science Practice 6: The student can work with scientific explanations and theories.

- 6.1: The student can *justify claims with evidence*.
- 6.2: The student can *construct explanations of phenomena based on evidence* produced through scientific practices.
- 6.3: The student can *articulate the reasons that scientific explanations and theories are refined or replaced*.
- 6.4: The student can *make claims and predictions about natural phenomena* based on scientific theories and models.
- 6.5: The student can *evaluate alternative scientific explanations*.

Science Practice 7: The student is able to connect and relate knowledge across various scales, concepts, and representations in and across domains.

- 7.1: The student can *connect phenomena and models* across spatial and temporal scales.
- 7.2: The student can *connect concepts* in and across domain(s) to generalize or extrapolate in and/or across enduring understandings and/or big ideas.

Appendix B

Table of Information and Equation Tables for AP Physics 1

ADVANCED PLACEMENT PHYSICS 1 TABLE OF INFORMATION, EFFECTIVE 2015

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	Coulomb's law constant, $k = 1/4\pi\varepsilon_0 = 9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$
Speed of light, $c = 3.00 \times 10^8$ m/s	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$

UNIT SYMBOLS	meter,	m	kelvin,	K	watt,	W	degree Celsius,	°C
	kilogram,	kg	hertz,	Hz	coulomb,	C		
	second,	s	newton,	N	volt,	V		
	ampere,	A	joule,	J	ohm,	Ω		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. Assume air resistance is negligible unless otherwise stated.
III. In all situations, positive work is defined as work done on a system.
IV. The direction of current is conventional current: the direction in which positive charge would drift.
V. Assume all batteries and meters are ideal unless otherwise stated.

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2} m v^2$$

$$\Delta E = W = F_{\parallel} d = Fd\cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(2\pi f t)$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp} F = rF\sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2} I \omega^2$$

$$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$$

$$U_s = \frac{1}{2} k x^2$$

$$\rho = \frac{m}{V}$$

- a = acceleration
- A = amplitude
- d = distance
- E = energy
- f = frequency
- F = force
- I = rotational inertia
- K = kinetic energy
- k = spring constant
- L = angular momentum
- ℓ = length
- m = mass
- P = power
- p = momentum
- r = radius or separation
- T = period
- t = time
- U = potential energy
- V = volume
- v = speed
- W = work done on a system
- x = position
- y = height
- α = angular acceleration
- μ = coefficient of friction
- θ = angle
- ρ = density
- τ = torque
- ω = angular speed

$$\Delta U_g = mg\,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

ELECTRICITY

$$\left|\vec{F}_E\right| = k\left|\frac{q_1 q_2}{r^2}\right|$$

$$I = \frac{\Delta q}{\Delta t}$$

$$R = \frac{\rho \ell}{A}$$

$$I = \frac{\Delta V}{R}$$

$$P = I\,\Delta V$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

- A = area
- F = force
- I = current
- ℓ = length
- P = power
- q = charge
- R = resistance
- r = separation
- t = time
- V = electric potential
- ρ = resistivity

WAVES

$$\lambda = \frac{v}{f}$$

- f = frequency
- v = speed
- λ = wavelength

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2}bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

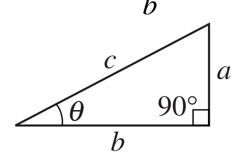
Rectangular solid
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

- A = area
- C = circumference
- V = volume
- S = surface area
- b = base
- h = height
- ℓ = length
- w = width
- r = radius

Right triangle
$c^2 = a^2 + b^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



Appendix C

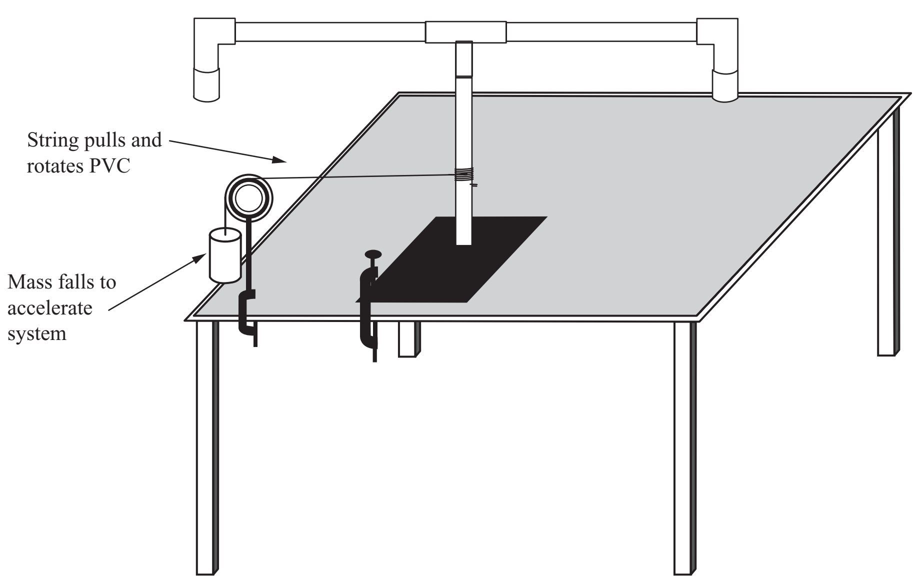
Lab — Determination of Rotational Inertia

Purpose: Design an experiment to determine the relationship between torque exerted on an apparatus and rotational inertia of the apparatus.

Overview for the teacher: For this lab, we recommend that students work in groups of four or five on each setup, which requires multiple sets of the materials listed below. Construct one setup and demonstrate briefly how pulling the string can cause the apparatus to rotate. Then give students the problem: "Design an experiment to determine the relationship between torque exerted on the apparatus and rotational inertia of the apparatus." Student designs may vary; for example, they may involve (a) varying torque by changing the size of the falling mass and then determining the rotational inertia of the apparatus, or (b) using a constant torque but varying the apparatus itself by using arms of different lengths or by adding objects to the end caps of the apparatus to vary the mass distribution. It is most useful to students if they are allowed to share their initial designs with the class to gain feedback prior to proceeding with the experiment. It is also useful to have a "post-lab peer review," where students then share their results with the entire class.

The materials listed below may not all be used by individual groups but will leave options for experimental method. Be willing to provide additional materials for students, depending upon their groups' designs — within reason. Students can be very creative!

Diagram of setup:



Materials needed for each setup:

- Approx. 1.5 m of $\frac{1}{2}$″ or $\frac{3}{4}$″ schedule 40 PVC (cut into five pieces)
- Two PVC 90° connectors
- One PVC straight T-connector
- Four PVC threaded end caps
- Four PVC right angles with one threaded end

- Small dense objects such as metal pellets, small metal nuts, or bolts
- Approx. 2 m sturdy string
- Ring-stand base
- Threaded rod to fit base
- Large C-clamp
- Duct tape
- Pulley and table clamp
- Calibrated masses
- Meterstick
- Stopwatch
- Padding to catch falling mass
- Motion sensor (optional, if available)
- Hot glue (optional)

Construction directions for the teacher:

1. Cut PVC into three short sections, about 30 cm each. The two horizontal sections must be of exactly equal length, and the vertical section can be shorter. Lengths are not crucial. Connect as shown above with the straight T-connector, the angled end connectors, and the end caps.
2. End caps can be filled with metal pellets or other small dense objects to give the apparatus a greater rotational inertia. (An extra set of end caps can be constructed with a different mass to vary the rotational inertia in another set of trials. Also, a second pair of longer or shorter PVC "arms" can vary the rotational inertia in yet another set of trials.)
3. Attach the threaded rod to the ring-stand base and slide the PVC apparatus over the vertical rod so that the T-connector rests on the top end of the threaded rod. (It may be necessary to shorten the vertical piece of PVC so that the T-connector sits on the top of the rod.)
4. Use a large C-clamp to secure the ring-stand base to the table so that it does not slip when the string pulls the apparatus to cause it to rotate.
5. Attach one end of the string securely onto the vertical PVC piece with duct tape so that it doesn't slip, and wrap the string around the pipe several times.
6. Attach the pulley to the table using the table clamp.
7. Run the free end of the string over the pulley and attach it to a known mass so that when the hanging mass is dropped, the PVC apparatus will rotate smoothly. It may be necessary to readjust lengths and connections until the apparatus is balanced and rotates smoothly.

Cautionary notes:

- For safety, hot glue or duct tape all the PVC connections to secure the apparatus as it rotates. If the apparatus is glued, students will need to be provided with multiple preprepared sets to switch out if they want to vary arm length or mass inside the arms.
- Students should be warned to place padding on the floor to catch the falling mass or provide a cage so the falling mass does not harm the motion sensor, if that method is used.
- It may be necessary to readjust lengths and connections so the apparatus rotates smoothly.

Suggested procedure (use only to prompt students as needed):

1. Place a motion sensor under the falling mass, and catch the mass before it hits the motion sensor. (If a motion sensor is unavailable, a stopwatch may be used to find time and distance, then calculate acceleration.) For best results, encourage students to use a large enough mass that the mass falls slowly and the apparatus rotates slowly.

2. If a motion sensor is used, determine the slope of the velocity-versus-time graph from the motion sensor to determine the acceleration of the falling mass, using a section of the graph that shows fairly uniform acceleration.
3. Construct a free-body diagram of the falling mass, and use the diagram to write a Newton's second law equation in the direction of motion.
4. From the mass and acceleration, determine the tension in the string.
5. Use the tension in the string and measurement of radius of the vertical PVC section to determine the torque exerted on the rotating apparatus to accelerate it.
6. Convert the linear acceleration of the string as it moves around the PVC to angular acceleration, using the radius of the PVC ($a = \alpha R$).
7. Now determine the moment of inertia of the rotating apparatus, using $\tau = I\alpha$.
8. Repeat the determination of rotational inertia using several different values for the falling mass.
9. Report an average value for rotational inertia and calculate percent difference.
10. Modify the apparatus, such as varying the lengths of the horizontal "arms" or changing the masses of the end caps, and repeat the experiment to determine what factors affect rotational inertia of such a device.
11. Discuss sources of uncertainty in measurements and/or select one measurement and "track" the uncertainty qualitatively through the calculations to determine the effect of the uncertainty on the final answer.
12. (As a challenge, encourage students to graph $m(g - a)R$ versus a/R and discuss the meaning of the slope and the intercept. The slope is the experimental value of rotational inertia, and the intercept is the frictional torque exerted by the support on the rotating apparatus.)

Appendix D

Class Activity — Determining the Angular Speed of a Fan

Purpose: As you demonstrate the strobing of a fan, student groups determine the angular speed of the blades and answer related questions.

Materials needed:

- Three-bladed fan
- Digital strobe light
- White correction fluid

Directions for the teacher:

1. Take the cover from the fan and make markings on the blades (separate markings on each blade or a single picture such as a face) with white correction fluid.
2. Leave the fan on a set speed.
3. In a darkened area, direct the strobe light toward the fan and away from students. Adjust the strobe frequency until the image on the fan blades appears to stop.
4. Show students other strobe rates that produce a standing image (i.e., strobe rates for which the fan frequency is a multiple of the strobe rate).
5. After the demonstration, divide students into small groups to do the related calculations and answer questions; then have a representative from each group report results.

Questions and calculations for student groups:

1. Record the strobe frequency (and convert to hertz [Hz], or cycles per second, if necessary). Keep in mind that the actual frequency of the fan could be an integral multiple of the strobe frequency, so double-check with higher strobe frequency multiples.
2. What happens to the image on the blades if the strobe frequency is slightly too high or too low? Why?
3. What happens if the rotation rate of the fan blades is a multiple of the strobe rate? Why?
4. What happens if the rotation rate of the fan blades is half of the strobe rate? Why?
5. Report the rotation rate in cycles per second.
6. How could you determine the linear speed of a point on the outside edge of a blade?
7. Why does the speed of the blades remain constant?

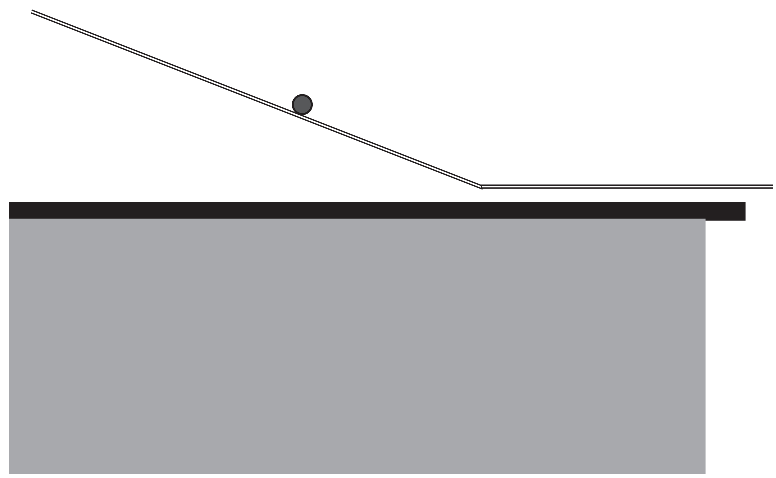
Teacher notes: This activity is best done as a demonstration because the fan blades are most visible with the cover of the fan removed, and a strobe light is safest when operated by you with proper warnings to students who might be affected by the flashing light. Involve students in predicting the answers to the questions and calculating rotation rate in rotations per second from counts per minute (if that is the value provided by the strobe). If the strobe rate is too high or too low, the students won't see distinguishable markings. When the image appears, the rotation rate of the fan may be a multiple of the strobe rate because the fan may make multiple turns between flashes of the strobe. If the rotation rate is half of the strobe rate, the blades will turn halfway around between each flash, so the image will appear to flip back and forth by 180 degrees. Students should report that the linear speed can be determined by measuring the radius of the blades and multiplying by that distance: $v = \omega R$.

Appendix E

Lab — Using Rotational Kinetic Energy for the Ball on a Ramp Lab

Purpose: Design an experiment that combines energy considerations with 2D kinematics to determine what fraction of the initial potential energy of a ball on a ramp is converted into rotational kinetic energy.

Diagram of setup:



Recommended materials for each setup:

- Steel ball
- Aluminum window track (or metersticks or narrow boards to create a groove for the ball to roll down a ramp)
- Ring stand and clamps (or books) to prop up ramp
- Protractor
- Carbon paper (optional, to make a mark on the floor where the ball lands)
- Masking tape or duct tape (to secure setup)
- Meterstick (for measurements)

Suggested procedure (use only to prompt students as needed):

1. Using the directions or a diagram provided by your teacher, design and build a ramp that can be used to release a ball, allowing it to roll down the ramp and onto the floor.
2. Sketch the setup in your journal and label all measurements, using the variables you will use to make calculations or discuss quantities.
3. Based on where the ball lands on the floor, find the speed v_1 with which the ball leaves the ramp. Several trials are recommended.
4. Calculate the speed v_{slide} with which the ball would leave the ramp *if* it slid instead of rolled down the ramp (e.g., if all potential energy was converted into *translational* kinetic energy).

5. Based on the difference between v_1 and v_{slide}, figure out what percentage of the system's initial potential energy converted into rotational as opposed to translational kinetic energy. This can be determined as follows:

   Since $U_{grav} = K_{trans} + K_{rot} = (\frac{1}{2})mv_{slide}^2$, with $K_{trans} = (\frac{1}{2})mv_1^2$, the fraction of the total kinetic energy that's rotational is

$$
\begin{aligned}
K_{rot}/K_{total} &= [K_{total} - K_{trans}]/K_{total} \\
&= [(\tfrac{1}{2})mv_{slide}^2 - (\tfrac{1}{2})mv_1^2]/[(\tfrac{1}{2})mv_{slide}^2] \\
&= [v_{slide}^2 - v_1^2]/[v_{slide}^2]
\end{aligned}
$$

Questions for students to incorporate into their analyses:

- What might be another method of determining the ball's velocity as it leaves the ramp?
- What are some possible sources of uncertainty?
- How might this apparatus be used to determine a value for coefficient of friction, if that value is not known?
- What ultimately happens to the energy of the ball when it hits the floor?
- How could the energy method be used to find the vertical velocity gained by the ball in its trip from the edge of the table to the floor?

Teacher notes and answers to lab analysis questions:

A derivation of $mgh = (\frac{7}{10})mv^2$, though not needed for this lab, could be shown to students after they complete this lab, though this derivation is outside the scope of Physics 1. Using the reasoning of the derivation, students could check how closely their calculated value of K_{rot}/K_{total} matches the theoretically expected value.

Other methods of measuring velocity might include (a) using the gravitational potential energy to predict velocity (in which case the role of rotational kinetic energy may not be as apparent), or (b) using a photogate at the bottom end of the ramp.

The mechanical energy "loss" from the ball/ramp system as the ball rolls down the ramp is due to friction, and the energy becomes thermal energy of molecules of both ball and ramp. When the ball hits the floor, the energy becomes thermal energy of molecules of both the ball and the floor.

Authors and Reviewers

Author

Connie Wells is a Physics and AP Physics teacher at Pembroke Hill School in Kansas City, Missouri. An AP Physics teacher since 1991, she has been active in test scoring and development for the College Board, serving on the AP Physics B and C Test Development Committee from 1997 to 2001. From 1995 to 2007, she served in various roles as Reader, Table Leader, and Question Leader for the AP Physics Reading. She is a teacher workshop and institute consultant for the College Board and is outgoing Chair of the Committee on Teacher Preparation for the American Association of Physics Teachers. Appointed by the College Board to the AP Physics Redesign Commission in 2006, Connie was Co-Chair of the AP Physics Curriculum Development and Assessment Committee and is currently Co-Chair of the AP Physics 2 Development Committee.

Co-Author

Andrew Elby is Associate Professor of Teaching and Learning, Policy and Leadership and Affiliate Associate Professor of Physics at the University of Maryland. A co-leader of the university's Physics Education Research Group, Andrew specializes in research, curriculum development, and teacher professional development aimed at incorporating authentic scientific inquiry into K–12 classrooms. Andrew is currently Chair of the Executive Council of the Physics Education Research Topical Group within the American Association of Physics Teachers. He was Co-Chair of the AP Physics Curriculum Development and Assessment Committee and is currently Co-Chair of the Physics 1 Development Committee.

Reviewer/Contributor

Mike Rulison, Oglethorpe University, Atlanta, Georgia

Reviewer

Peggy Bertrand, University of Tennessee, Knoxville, Tennessee

Paul Lulai, St. Anthony Village High School, St. Anthony, Minnesota

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