CollegeBoard AP®

SYLLABUS DEVELOPMENT GUIDE

AP® Calculus BC

The guide contains the following sections and information:

Curricular Requirements

The curricular requirements are the core elements of the course. A syllabus must provide explicit evidence of each requirement based on the required evidence statement(s). The Unit Guides and the "Instructional Approaches" section of the *AP® Calculus AB and BC Course and Exam Description* (CED) may be useful in providing evidence for satisfying these curricular requirements.

Required Evidence

These statements describe the type of evidence and level of detail required in the syllabus to demonstrate how the curricular requirement is met in the course.

Note: Curricular requirements may have more than one required evidence statement. Each statement must be addressed to fulfill the requirement.

Clarifying Terms

These statements define terms in the Syllabus Development Guide that may have multiple meanings.

Samples of Evidence

For each curricular requirement, three separate samples of evidence are provided. These samples provide either verbatim evidence or descriptions of what acceptable evidence could look like in a syllabus.

Curricular Requirements

CR1	The students and teacher have access to a college-level calculus textbook, in print or electronic format.	*See page:* 3
CR2	The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.	*See page:* 4
CR3	The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes.	*See page:* 6
CR4	The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations.	*See page:* 7
CR5	The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification.	*See page:* 9
CR6	The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation.	*See page:* 10
CR7	Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.	*See page:* 11
CR8	The course provides opportunities for students to use calculus to solve real-world problems.	*See page:* 13

Curricular Requirement 1

The students and teacher have access to a college-level calculus textbook, in print or electronic format.

Required Evidence

- ☐ The syllabus must list the title, author, and publication date of a college-level calculus textbook.

Samples of Evidence

1. *Calculus of a Single Variable* (Larson, Hostetler, Edwards, 8th Edition, 2005).

2. The syllabus includes the author, title, and publication date of a college-level textbook and states that each student has access to an electronic version of the textbook.

3. The syllabus states the title, author, and publication date of a textbook from the example textbook list published by the College Board on the AP Calculus BC Course Audit page on AP Central.

Curricular Requirement 2

The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.

Required Evidence

- ☐ The syllabus must include an outline of course content by unit title or topic using any organizational approach with the associated big idea(s) to demonstrate the inclusion of required course content. All three big ideas must be included: Change, Limits, and Analysis of Functions.

Note: If the syllabus demonstrates a different approach than the units outlined in the AP Course and Exam Description (CED), the teacher must indicate where the content and big ideas of each unit in the CED will be taught.

Samples of Evidence

1. The AP Calculus BC syllabus includes a list of the following units listed in the AP Course and Exam Description (CED):
    - Unit 1: Limits and Continuity
    - Unit 2: Differentiation: Definition and Fundamental Properties
    - Unit 3: Differentiation: Composite, Implicit, and Inverse Functions
    - Unit 4: Contextual Applications of Differentiation
    - Unit 5: Analytical Applications of Differentiation
    - Unit 6: Integration and Accumulation of Change
    - Unit 7: Differential Equations
    - Unit 8: Applications of Integration
    - Unit 9: Parametric Equations, Polar Coordinates, and Vector-Valued Functions
    - Unit 10: Infinite Sequences and Series

    Note that the big idea of Limits specifically appears in Units 1, 2, 5, 6, 8, and 10; the big idea of Change specifically appears in Units 2, 4, 5, 6, 7, 8, 9, and 10; and the big idea of Analysis of Functions appears in Units 3, 4, 5, 6, 7, 8, 9,and 10.

2. This AP Calculus BC course follows the units as presented in the CED, except that topics from Unit 9 of the CED are incorporated with other units of study:
    - Unit 1: Limits and Continuity
    - Unit 2: Differentiation: Definition and Fundamental Properties building on the big ideas of limit and rate of change
    - Unit 3: Differentiation: Composite, Implicit, and Inverse Functions, involving analysis of functions and rate of change
    - Unit 4: Contextual Applications of Differentiation involving rate of change, analysis of functions and limits
    - BC Mini-Unit on Differentiation [Topics 9.1, 9.2, 9.4, and 9.7]
    - Unit 5: Analytical Applications of Differentiation and extension to parametric, polar, and vector-valued functions involving rate of change, analysis of functions, and limits
    - Unit 6: Integration and Accumulation of Change involving limits and analysis of functions as well as change in accumulation

- Unit 7: Differential Equations involving analysis of functions, change, and limits
- Unit 8A: Applications of Integration [Topics 8.1, 8.2, 9.5, and 9.6] involving change and analysis of functions
- Unit 8B: Applications of Integration [Topics 8.3, 8.4, 8.5, 8.6, 9.8, and 9.9] involving change and analysis of functions
- Unit 8C: Applications of Integration [Topics 8.7, 8.8, 8.9, 8.10, 8.11, 8.12, 8.13, and 9.3] involving change and analysis of functions
- Unit 10: Infinite Sequences and Series involving rate of change, analysis of functions, and limits

3. The big idea of Limits is involved in:

   Chapter 1, limits and continuity; Chapter 2, defining the derivative and developing the basic rules and properties; Chapter 4, applying the derivative analytically and contextually; Chapter 5, introducing L'Hospital's rule; Chapter 6, integration as accumulation of change and the Fundamental Theorem of Calculus; and Chapter 10, infinite sequences and series.

   The big idea of Change appears in:

   Chapter 2, defining derivative and developing the properties of a derivative; Chapter 4, applying the derivative in different contexts including parametric equations; Chapter 6, integration as accumulation and the Fundamental Theorem of Calculus; Chapter 7, applications of integration including polar coordinates and vector-valued functions.

   The big idea of Analysis of Functions appears in:

   Chapter 2, defining derivative and developing the properties of a derivative; Chapter 3, differentiation of composite, inverse and implicit functions; Chapter 5, applying the derivative in different situations; Chapter 6, integration as accumulation of change and the Fundamental Theorem of Calculus; Chapter 7, Applications of integration including polar coordinates and vector-valued functions; Chapter 8, differential equations.

Curricular Requirement 3

The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes, as outlined in the AP Course and Exam Description (CED).

Required Evidence

- ☐ The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students use two or more skills under Mathematical Practice 1. The lessons, activities, or assignments must be described or identified so that the corresponding skill(s) are evident.

  AND

- ☐ One of those lessons, activities, or assignments must incorporate the portion of Skill 1.E in which students apply appropriate mathematical rules or procedures **without** technology.

  It is not necessary that the skills appear in a single lesson, activity, or assignment.

Samples of Evidence

1. On a worksheet with functions described graphically, analytically, and numerically, students use Euler's method to approximate a value of $f(x)$. They are then asked to discuss how the sign of $f''(x)$ affects whether their approximation is an overestimate or an underestimate. For the first function, students need to approximate $f(x)$ using Euler's method starting at $x = 1$ and going to $x = 2$ using 10 subintervals of equal width. (1.E, 1.F)

2. The syllabus includes an assignment in which students are given a set of functions expressed analytically and asked to identify which are composite functions, then apply the chain rule to differentiate them manually. (1.C, 1.E)

3. The syllabus includes a homework problem set that asks students to identify the appropriate integration techniques to apply to each of several definite integrals, compute the antiderivatives manually, and then check their results using technology. In pairs, they then discuss any of their results that do not match the results when using the technology. (1.C, 1.E)

Curricular Requirement 4

The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations, as outlined in the AP Course and Exam Description (CED).

Required Evidence

- ☐ The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students work with multiple representations. Each of the four representations (analytical, numerical, graphical, and verbal) must be in at least one of the provided lessons, activities, or assignments. It is not necessary that all four representations appear in a single lesson, activity, or assignment.

  AND
- ☐ There must be evidence of a connection between at least two different representations in at least one of the provided lessons, activities, or assignments aligned with Skills 2.C, 2.D, or 2.E.
- ☐ The lessons, activities, or assignments must be described or identified so that the skill(s) and representations are evident.

Clarifying Terms

Representations – Analytical: one that is given as an algebraic expression or equation.

Representations – Numerical: one whose values are given as tabular data or sets of ordered pairs.

Representations – Graphical: one that is drawn as a set of points on a pair of coordinate axes or in which information is presented in a diagram.

Representations – Verbal: one that is described in words rather than mathematical symbols.

Samples of Evidence

1. Students work on an assignment where they are given a function written analytically representing a population in terms of time and asked to graph the function, then to construct a table of values and use the table to estimate the area of the region bounded by the function, the t-axis, $t = 2$, and $t = 6$ using midpoint Riemann sums with four subintervals of equal length. They are asked to discuss how the Riemann sum is connected to the context of the problem. (2.B, 2.D)

2. Students are given an assignment in which they are asked to write a third-degree Taylor polynomial for several different functions f. In one case, information about the first, second, and third derivatives of f is given in a table, in another case, some of the information is presented graphically; in another case the function f is given analytically. Students are asked to explain how the signs of the values in the table are connected to the shape of the graphs of the polynomials. (2.B) In a separate assignment, students are given verbal descriptions of a function f such as "the first derivative is negative and increasing" and are asked to sketch a possible graph of f. (2.D)

3. The syllabus describes a group activity in which two small pieces of paper are prepared for each of several functions, one that displays the graph of the function and the other that shows a well-labeled sign chart for the first derivative of the function over a relevant interval. Each student gets a card, and students walk around the room, talking to their classmates until they find their match. Then each pair of students

writes on the board a verbal description of the graph of the second derivative of the original function and the verbal description of the second derivative. (graphical, verbal) (2.B, 2.C, 2.E)

Students are given an assignment in which they are given analytical representations of the derivatives of several functions, defined over a given interval, and asked to create a candidate's table of possible values of x and $f(x)$ to find the absolute maximum value of the function over the interval. They must describe in writing the connection between the table of values and the graph of the function. (analytical, graphical, numerical) (2.B, 2.D)

Curricular Requirement 5

The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification, as outlined in the AP Course and Exam Description (CED).

Required Evidence

- ☐ The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students use two or more skills under Mathematical Practice 3. The lessons, activities, or assignments must be described or identified so that the corresponding skill(s) are evident.

  AND

- ☐ One of those skills must be 3.C, "Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied."

  AND

- ☐ One of those skills must be either 3.E, "Provide reasons or rationales for solutions and conclusions," or 3.F, "Explain the meaning of mathematical solutions in context."

  It is not necessary that the skills appear in a single lesson, activity, or assignment.

Clarifying Terms

"Solutions and Conclusions": any answer based on the application of a definition, theorem, or test.

Samples of Evidence

1. The syllabus includes a formative assessment question that gives students several functions described analytically on specified domains. The question asks students to determine whether each satisfies the hypotheses of the Mean Value Theorem, Extreme Value Theorem, and the Intermediate Value Theorem, as well as to provide written reasons for their choices. (3.C, 3.E)

   In another activity, the students are given information about a function and selected values of the function in a table. They are asked questions like, "Is there a value of x between a and b such that $f(x) = k$ or such that $f'(x) = m$?" The students are asked to find reasons for these conclusions. (3.E)

2. In a class activity, students work in groups to decide whether the conditions for finding an error bound using the alternating series test are met for a variety of different series. (3.C) If the conditions are met, the students find the error bound and explain what this means in the context of the problem (3.F); if the conditions are not met, the students describe what might be changed in the given information so the conditions would be met.

3. The syllabus includes an instructional activity in which students are given a graph of f' and the graph of f''. Based on information found in these graphs, students work individually to identify local extrema for f and write justifications. Then students compare their work with a neighbor's, explaining their reasoning to each other (3.E), and both make refinements. Finally, the class works together to develop a clear and complete statement of both the first derivative test and the second derivative test based on what they have concluded from these examples. As an assessment of whether the students have learned the necessary concepts, they are given an exam question that asks them to determine whether the second derivative test applies to a variety of functions at specific x-values (3.C).

Curricular Requirement 6

The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation, as outlined in the AP Course and Exam Description (CED).

Required Evidence

- ☐ The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students are given the opportunity to communicate their understanding of calculus concepts, processes, or procedures using appropriate mathematical language. (Skill 4.A)

  AND

- ☐ The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students demonstrate notational fluency by either connecting different notations for the same concept or using appropriate mathematical notation in applying procedures. (Skill 4.C)
- ☐ The lessons, activities, or assignments must be described or identified so that the corresponding skill(s) are evident. It is not necessary that the skills appear in a single lesson, activity, or assignment.

Samples of Evidence

1. Students are given a worksheet with two columns where they match equivalent forms of notation for calculus concepts, such as the limit of a Riemann sum with the number of subdivisions tending to infinity and the corresponding definite integral; limit of a difference quotient and the corresponding derivative at a specific x-value; two equivalent forms of the limit of a difference quotient; etc. (4.C) After students have worked individually, they share their answers with a partner, explaining their reasoning for each of the matches, and then each pair orally explains to the class their reasoning for one match with the instructor checking that they are using appropriate mathematical language. (4.A)

2. The syllabus describes an assignment in which students are given various real-world situations described in words that they must translate into equivalent differential equations. (4.C) Each student then picks one of these situations and writes a sentence on how they translated the description into a differential equation; the teacher grades these sentences for the use of appropriate mathematical language. (4.A)

3. For homework, students are asked to write a paragraph describing the difference between series that converge conditionally, series that converge absolutely, and series that diverge and to give an example of each, using correct notation. The next day, they exchange their paragraphs with another class member and write at least three sentences that begin with one of the following sentence starters: "I agree with you that … because …." or "I did not understand the part where you …." Or "I wonder if the example … works because …." (4.A) As a follow-up assignment, students are given various series written analytically not in summation notation and must rewrite each in proper summation notation (4.C) and then decide if each converges conditionally, converges absolutely, or diverges.

Curricular Requirement 7

Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.

Required Evidence

- ☐ The syllabus includes a statement that each student has individual access to an approved graphing calculator.

  AND

- ☐ The syllabus must include a description or copy of one or more lessons, activities, or assignments in which students use graphing calculators to:
  - graph functions
  - solve equations
  - perform numerical differentiation
  - perform numerical integration
  - explore or interpret calculus concepts

  It is not necessary that the above requirements appear in a single lesson, activity, or assignment.

Samples of Evidence

1. The syllabus indicates that every student has a graphing calculator. If any students do not have their own, the mathematics department has ones that can be assigned to those students for the school year, to be returned at the end of the course.

   The syllabus describes an activity designed to help students become familiar with some of the calculator features needed for the AP Exam. In this activity, the students are given two functions expressed analytically. They are asked to use the calculator's equation solver to determine where the functions intersect, then to use the calculator to draw a graph of the functions and use the graph to confirm that they do intersect at the points found earlier, and finally to use the calculator's numerical differentiation feature to approximate the derivative of each function at the point(s) of intersection.

   The syllabus also describes an assessment in which students are given various functions that they cannot integrate analytically using the techniques they have learned. They must instead use the graphing calculator's numerical integration feature to compute definite integrals of these functions on stated intervals.

   The syllabus also includes a worksheet that asks students to find the first derivative of several functions analytically by hand and determine at which x-value(s) each derivative does not exist. The worksheet then asks the students to interpret the results they found manually by graphing each of these functions on the calculator in order to see why the derivative does not exist at the x-value(s) they found.

2. The syllabus states that each student should purchase a graphing calculator before the course begins and that the department will provide a loaner calculator to any student who does not purchase one. In a class activity, students are placed into groups of two or three and given three functions: $f(x) = \sin x$, $g(x) = \cos x$, and $h(x) = e^x$. For each function, they are asked to compute several Taylor polynomials (centered at $x = 0$) by hand and use their calculator to draw the graphs. They will reproduce the graphs (the given function and each of the polynomials in a different colored pencil)

and discuss how close each approximation is to the graph of the given function. Each piece of graph paper will have the graph of one of the functions and the graphs of the polynomial approximations for that function.

1. For $f(x) = \sin x$ they will find polynomials of degree 1, 3, and 5.
2. For $g(x) = \cos x$ they will find polynomials of degree 0, 2, and 4.
3. For $h(x) = e^x$ they will find polynomials of degree 0, 1, 2, and 3.

Note: The students will determine a window for each function that allows them to see enough of each graph to determine an interval on which the polynomial would be a good approximation and to see and be able to discuss where the polynomials start straying from the function.

The students will then use the calculator's numerical differentiation feature to compare the value of the slopes of the three given transcendental functions to the values of the slopes of the polynomials at selected-values.

The syllabus describes another activity in which students use their graphing calculators to explore the convergence and divergence of improper integrals and an activity in which students use their calculators to find the intersection point of two polar curves and then find the area between the two curves by using the calculator's numerical integration feature.

3. The syllabus states that each student in the course owns a graphing calculator or has made arrangements to check one out of the library for the year.

   The syllabus includes an activity that asks students to use the calculator to graph the derivative of several functions in order to make a conjecture about the number of critical values of the functions and whether those values are relative maximums, relative minimums, or neither. The syllabus then asks the students to use the calculator's equation solver to approximate each zero correct to three decimal places.

   The syllabus includes another activity that asks students to write out the work of computer the derivatives of several functions manually using correct notation and then to use the calculator's numerical differentiation feature to approximate the derivatives of these functions at specified x-values.

   The syllabus also includes a take-home assignment that asks students to use the calculator to explore what happens to the values of left-hand and right-hand Riemann sums as the number of subdivisions increases.

   The syllabus also describes an exam that includes several old free-response questions that require students to use the calculator's numerical integration feature to compute the area between two polar curves correct to three decimal places.

Curricular Requirement 8

The course provides opportunities for students to use calculus to solve real-world problems.

Required Evidence

- ☐ The syllabus must provide a description of one or more lessons, activities, or assignments requiring students to apply their knowledge of AP Calculus concepts to solve real-world problems.

Samples of Evidence

1. The syllabus describes a group activity that asks students to record speeds for a radio-controlled vehicle at specified intervals. Students then use Riemann sums to estimate the distance the vehicle has traveled.

2. Students are given five real-world free-response questions (FRQs) from released AP Exams to work on as a review for the AP Exam. Questions are then reviewed in class, with different students presenting their answers on the board. Emphasis will be placed on writing excellent FRQ answers using full sentences, justifying their answers, and using correct mathematical notation.

3. The syllabus includes an activity in which students are given various real-world geometric series problems (such as the amount of medicine in a patient's bloodstream or the present value of a future stream of income distributed over time) that they must use calculus techniques to solve.