SAMPLE SYLLABUS #1

AP® Calculus AB

Curricular Requirements

CR1	The students and teacher have access to a college-level calculus textbook, in print or electronic format.	*See page:* 2
CR2	The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description.	*See page:* 3
CR3	The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes.	*See pages:* 4, 7, 8, 9
CR4	The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations.	*See pages:* 3, 4, 7, 10, 11, 15, 16
CR5	The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification.	*See pages:* 4, 5, 6, 9, 10
CR6	The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation.	*See pages:* 5, 6, 8, 9, 14
CR7	Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts.	*See pages:* 2, 5, 6, 9, 14, 17
CR8	The course provides opportunities for students to use calculus to solve real-world problems.	*See pages:* 3, 5, 9, 18

Advanced Placement Calculus AB Sample Syllabus #1

Course Overview

Course Overview: AP® Calculus AB is equivalent to a first-semester college calculus course. Topics include functions, limits and continuity, derivatives, and integrals. The course will focus on applying the skills and concepts of calculus to modeling and solving problems across multiple representations.

Course Expectations

Students are expected to complete all homework problems to the best of their ability. If they need additional support, they can refer to the additional resources listed below.

The Personal Progress Checks (PPC) that are assigned online for this course through the student's College Board account are to be completed on time; exceptions will not be made.

Students will take daily quizzes. These quizzes are short and are intended to check for understanding of concepts and skills that were recently taught. Students are required to make all corrections when the quizzes are returned to them.

All projects are due by the indicated due date.

Technology Requirement

Students will be provided with a TI-Nspire graphing calculator. Some problems throughout the course will require them to use their graphing calculators. **CR7**

Textbook Requirement

Sullivan, Michael, and Kathleen Miranda. *Calculus* (for the AP Course), 2nd ed. (New York: Bedford, Freeman & Worth, 2017) **CR1**

Additional Resources

Students can watch a video on my YouTube channel corresponding to the lesson we covered in class. On a regular basis, I send a video link to remind students of this resource.

- Students can log in to Davidson Next for AP Calculus AB. Students will find video lessons for the topic we are going over in class as well as practice problems.
- Students can log in to the website "GetAFive" using the instruction sheet provided. This site has videos and problems grouped according to topic.
- Students have the option of coming to me for help before or after school.

Course Outline and Pacing – Starting School After Labor Day

- September/October – Unit 1
- October/November – Units 2 and 3
- November/December – Unit 4
- December/January – Unit 5

CR7
The syllabus includes a statement that each student has individual access to an approved graphing calculator.

AND

The syllabus must include a description of at least one activity in which students use graphing calculators to:

- graph functions
- solve equations
- perform numerical differentiation
- perform numerical integration
- explore or interpret calculus concepts

CR1
The syllabus must list the title, author, and publication date of a college-level calculus textbook.

- January/February – Unit 6
- February/March – Unit 7
- March/April – Unit 8
- April/May – AP Review

Student Practice

Throughout each unit, **Topic Questions** will be provided to help students check their understanding. The Topic Questions are especially useful for confirming understanding of difficult or foundational topics before moving on to new content or skills that build upon prior topics. Topic Questions can be assigned before, during, or after a lesson, and as in-class work or homework. Students will get rationales for each Topic Question that will help them understand why an answer is correct or incorrect, and their results will reveal misunderstandings to help them target the content and skills needed for additional practice.

At the end of each unit or at key points within a unit, **Personal Progress Checks** will be provided in class or as homework assignments in AP Classroom. Students will get a personal report with feedback on every topic, skill, and question that they can use to chart their progress, and their results will come with rationales that explain every question's answer. One to two class periods are set aside to re-teach skills based on the results of the Personal Progress Checks.

Course Outline and Description: CR2

Unit 1: Limits and Continuity (Big Ideas: Change, Limits, Analysis of Functions)

1.1 Introducing Calculus: Can Change Occur at an Instant? (Skill 2.B)

In a classroom activity, students will calculate the velocities (the average rate of change) of several automobiles using both functions given analytically and data presented in a table of time versus displacement. Students will use their information to approximate the instantaneous velocity of the automobile at a particular time t and to sketch a graph of velocity as a function of time. They will provide a verbal (that is, written in words) interpretation of the movement of each vehicle (such as "The car's velocity is positive and decreasing") and explain how their verbal interpretation is connected to the graph they have drawn. CR4 CR8

1.2 Defining Limits and Using Limit Notation (Skill 2.B)

In a classroom activity, students will sort cards pertaining to the graph of a function f consisting of vertical asymptotes, horizontal asymptotes, jump, removable, and non-removable discontinuities. Students will have to match selected portions of the graph to its written description and symbolic (notation) description. Here, students are learning how to express limits in both written and symbolic form to understand the behavior of a function f as f gets sufficiently close to a particular x-value.

1.3 Estimating Limit Values from Graphs (Skill 2.B)

In a classroom activity, students will work in pairs to use a graph of a function to approximate the value of a limit, if it exists. Students will use the strategy of Concepts with Color, located on page 204 in the CED, where one student will trace the graph of the function from the left in one color while the other student will trace the graph from the right using another colored pencil. Then, using correct language to describe a limit, students will explain whether or not the limit exists.

1.4 Estimating Limit Values from Tables (Skill 2.B)

In a homework assignment, students will complete a table of values to find the limit, if it exists, for a set of functions. In some of the problems, a graphing calculator will be required. Students may notice in some problems that direct substitution would

CR2
The syllabus must include an outline of course content by unit title or topic using any organizational approach with the associated big idea(s) to demonstrate the inclusion of required course content. All three big ideas must be included: Change, Limits, and Analysis of Functions.

CR4
The syllabus must include a description of at least one activity in which students work with multiple representations. Each of the four representations (analytical, numerical, graphical, and verbal) must be in at least one of the provided activities.

AND

There must be evidence of a connection between at least two different representations in at least one activity, aligned with Skills 2.C, 2.D, or 2.E.

The activity or activities must be labeled with the corresponding skill(s).

CR8
The syllabus must provide a description of one or more activity requiring students to apply their knowledge of AP Calculus concepts to solve real-world problems.

have worked, while in other problems, direct substitution does not work, but the problem still has a limit. A problem where direct substitution fails but still has a limit gets the student to think about how else they could come up with the answer without using technology. (Getting them prepared to think about using algebra.)

1.5 Determining Limits Using Algebraic Properties of Limits (Skill 1.E)

Students will complete a homework assignment applying the Algebraic Properties of Limits across multiple representations. Students will be given information about the graph of function f, a polynomial function g expressed symbolically, a rational function h expressed symbolically, a table of values for a function k, and a written description of the limits for functions r and s. Although all functions may not be used in one problem, each limit problem will consist of at least two different representations, and students will be asked to explain how those representations are connected. In addition to finding limits across multiple representations, students will discover in a problem or two that although the limit of a function f and the limit of a function g may not exist, the limit of $f + g$, does exist. **CR4**

1.6 Determining Limits Using Algebraic Manipulation (Skill 1.C)

Students will complete a homework assignment where they be given limits of various functions expressed analytically. The students will have to identify the appropriate mathematical procedure (including direct substitution, factoring, finding a common denominator, multiplying by a conjugate, and rewriting the expression) and then implement that procedure to compute the limit. **CR3**

1.7 Selecting Procedures for Determining Limits (Skill 1.C)

Students will complete an activity where they have to choose a method for determining a limit arranged in a chart. They will start with direct substitution; if they get 0/0, they will have to choose from Algebra, Table of Values, or a Graph as a means for finding the limit. Then, they will write a brief explanation why they chose that method for finding the limit. Students will also use a flow chart to help them find limits. **CR3**

☐ Complete Personal Progress Check MCQ Part A for Unit 1

1.8 Determine Limits Using the Squeeze Theorem (Skill 3.C)

Students will complete a three-part homework assignment using the Squeeze Theorem. For each part, students will have to decide if the conditions of the Squeeze Theorem are met and, if so, provide the reasoning for their claim that the conditions are satisfied and then proceed to use the theorem to find the indicated limit. The first part of the worksheet will consist of graphs where students have to decide if the conditions are met. In the second part, a function will be sandwiched between two other functions and students will have to check if the conditions are met before finding the limit. In the third part, the students will be given a function where they will have to sandwich the function between two values and proceed from there in trying to find the limit. **CR5**

1.9 Connecting Multiple Representations of Limits (Skill 2.C)

Students will complete a homework assignment to review the limits they've studied so far. This assignment will be broken into parts: in part one, students will use a graph to find the limit; in part two, students will use a table of values to find the limit; in part three, students will use algebra to find the limit. In the final part, students will have to use multiple representations to find a limit. The representations will include two graphs, two functions, and a table of values. **CR4**

1.10 Exploring Types of Discontinuities (Skill 3.B)

Students will complete an activity in class where they will learn the different types of discontinuities. In one part, students will complete a chart using the given graph of a function. The columns of the chart will consist of finding $f(a)$, $\lim_{x\to a+} f(x)$, $\lim_{x\to a-} f(x)$, $\lim_{x\to a} f(x)$, and whether $f(a) = \lim_{x\to a} f(x)$. The students will

CR3
The syllabus must include a description of one or more activities in which students use two or more skills under Mathematical Practice 1. The activity must be labeled with the corresponding skill(s).

AND

One of those activities must incorporate the portion of Skill 1.E in which students apply appropriate mathematical rules or procedures without technology.

CR5
The syllabus must include a description of one or more activity in which students use two or more skills under Mathematical Practice 3. The activity or activities must be labeled with the corresponding skill(s).

AND

One of those skills must be 3.C.

AND

One of those skills must be either 3.E or 3.F.

learn about three types of discontinuities by completing this table – removable, jump (piecewise), and asymptotic. They will also justify the type of discontinuity using correct notation. We will also refer back to the activity in Topic 1.2. **CR6**

1.11 Defining Continuity at a Point (Skill 3.C)

After students complete the activity from 1.10, they will learn what conditions are required for a function to be continuous at a point. Students have a tendency to give weak explanations for justifying whether a function is continuous at a point or not. They fail to use proper notation and need practice applying the definition of continuity to problems in a variety of representations. Also, to help students achieve better communication and notational fluency with the definition of continuity, I will use a classroom activity that includes error analysis. Students will critique student samples from prior FRQ's that either correctly or incorrectly used the definition of continuity in justifying answers. We will also refer back to the activity in Topic 1.2. **CR5**

☐ Complete Personal Progress Check MCQ Part B for Unit 1

1.12 Confirming Continuity over an Interval (Skill 1.E)

Students will complete a homework assignment where they have to check for continuity over different types of intervals, i.e., closed, open, half-open, etc. Problems will consist of functions that are not continuous at an interior point of an interval, endpoint of an interval, and at some point where no interval is given. Problems will also consist of functions that are continuous on the given interval. Piecewise functions will be emphasized in this assignment because students fail to check for continuity where the domain is broken up. Confirming continuity is an essential condition for Existence Theorems.

1.13 Removing Discontinuities (Skill 1.E)

Students will complete a homework assignment consisting of problems where a function is not continuous at a point but the problem can be rewritten or extended so that the function is now continuous at that point.

1.14 Connecting Infinite Limits and Vertical Asymptotes (Skill 3.D)

Using a table of values for x, students will use a calculator to find values for a given function $f(x)$. They will notice that the values for $f(x)$ either approach positive or negative infinity. Then students will use their graphing calculator to explore the graph of the function so that they could verify the location of the vertical asymptote. Using the table of values, students will use limit notation to explain why the function has a vertical asymptote near that value of x. We will also refer back to the activity in Topic 1.2. **CR7**

1.15 Connecting Limits at Infinity and Horizontal Asymptotes (Skill 2.D)

Students will complete an activity broken into three parts. In the first part, students will indicate what the y-values of a function are approaching as the x - values approach positive or negative infinity. In the second part, students will use technology to graph a given function and use their graph to determine the equation of the horizontal asymptote. In the third part, students will determine the horizontal asymptotes without technology by using the information they obtained in parts one and two. Students will have to make the connection in parts one and two in order to answer part three without technology. However, students may use technology to confirm the horizontal asymptotes of a function in part three. We will also refer back to the activity in Topic 1.2. **CR8**

☐ Complete Personal Progress Check FRQ A for Unit 1

1.16 Working with the Intermediate Value Theorem (Skill 3.E)

Students will complete an activity using the Intermediate Value Theorem. In order to apply the IVT, students must address the essential condition of continuity on a closed interval. In part one, students will use the strategy of sentence starters, indicated in the CED on page 212, to check for continuity on a closed interval.

CR6

The syllabus must include a description of at least one activity in which students are given the opportunity to communicate their understanding of calculus concepts, processes, or procedures using appropriate mathematical language. (Skill 4.A)

AND

The syllabus must include a description of at least one activity in which students demonstrate notational fluency by either connecting different notations for the same concept or using appropriate mathematical notation in applying procedures. (Skill 4.C)

The activity or activities must be labeled with the corresponding skill(s).

In part two, students will use a template to write an argument using IVT. The problems in part two will include a variety of contexts in which students have to apply IVT. **CR5**

- ☐ Take Unit 1 Test.

Unit 2: Differentiation: Definition and Fundamental Properties (Big Ideas: Change, Limits, Analysis of Functions)

2.1 Defining Average and Instantaneous Rates of Change at a Point (Skill 2.B)

In a class activity, students will use the graph of a function to find the average rate of change (the slope of the secant line) of a function over several closed intervals. Then, students will approximate the instantaneous rate of change at a point (the slope of the tangent line) using their average rates of change. The class activity will consist of several representations from past FRQ's consisting of tables of values, graphs, and quantities modeled by a function. Units will be required.

2.2 Defining the Derivative of a Function and Using Derivative Notations (Skills 1.D and 4.C)

Using the activity from 2.1 as a reference, students will learn the definition of derivative in three different forms.

In a homework assignment, students will use all three forms to find the derivative of a function. On this same assignment, students will be given limits of various difference quotients with the denominator approaching 0 and will be asked to identify each as the derivative of a specific function at a specific x-value; for each limit, the students will write in appropriate mathematical language an explanation of how they determine what derivative the limit represents. **CR6**

2.3 Estimating Derivatives of a Function at a Point (Skill 1.E)

In a class exercise, students will estimate the derivative of a function at a point using a table of values.

The class activity will consist of several representations from past FRQ's where students will need to show a difference quotient and attach units to their answer.

2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist (Skill 3.E)

In a class activity, students will learn to determine if a function is a differentiable or not. At this point in the course, students interpret a function being differentiable at a point $x = a$ as the slope of the tangent line existing at $x = a$ In the first part of the activity, students will be given the graphs of two functions; one graph is of a continuous function, and the other graph is a function that is not continuous at a given point. Students will draw several tangent lines along the graphs of both functions using a straight edge. Then, students will use the strategy of turn and talk to discuss which functions are differentiable and why. They will discuss the essential condition for differentiability. After students learn that continuity is a requirement for differentiability, they will complete the second part of the activity, where they will consider graphs of continuous functions and draw tangent lines at various points along their graphs. The students should discover that slopes of tangents do not exist at corner points, cusps, or vertical lines. **CR6**

2.5 Applying the Power Rule (Skill 1.E)

In a class activity, students will use their graphing calculator to discover the power rule for derivatives. Students will enter functions such as, $y = x$, $y = x^2$, $y = x^3$ into their calculators and graph the derivatives of the functions one at a time in order to explore the graphs and make a conjecture about the derivative of a power function. Then, students will use the strategy of turn and talk to try and generalize a rule for finding the derivative of a power function. **CR7**

- ☐ Complete Personal Progress Check MCQ A for Unit 2

2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple (Skill 1.E)

Students will complete a homework assignment applying the power rule to equations of the form x^n.

In numerous problems, students will have to perform algebra first essentially rewriting functions involving products and quotients using algebra, before applying the power rule. **CR3**

2.7 Derivatives of cos x, sin x, e^x, and ln x (Skill 1.E)

In several class activities, students will discover the derivative rules for these functions in multiple ways. First, as a class, we will derive the derivative of cos x using the definition of derivative. Second, students will graph one period of the sine curve and then draw various tangent lines at selected values of x to sketch the derivative. Third, students will explore the derivatives of e^x and ln x by using their graphing calculator. Finally, students will have to recognize the limit expression as the definition of derivative. **CR4**

2.8 The Product Rule

In a homework assignment, students will use the product rule to find the derivative involving the functions listed in the topics above. The assignment will consist of the following parts. In part one, students will find the derivative given the equation of a function. In this part, the derivative of some problems should use algebra first and would not necessarily warrant a product rule even though the function is a product. In part two, students will use a table of values to find the value of the derivative. Students will have to pull values from the table in order to compute their answer. In the third part, students will use the graphs of functions to find the value of the derivative. (Skill 1.E)

2.9 The Quotient Rule (Skill 1.E)

In a homework assignment, students will use the quotient rule to find the derivative involving the functions listed in the topics above. The assignment will be cumulative involving all the other derivative rules, but an emphasis will be on the quotient rule. In part one, students will find the derivative given a symbolic representation of a function. In this part, the derivative of some problems should use algebra first and would not necessarily warrant a quotient rule even though the function is a quotient. In part two, students will use a table of values to find the value of the derivative. Students will have to pull values from the table in order to compute their answer. In the third part, students will use the graphs of functions to find the value of the derivative.

- ☐ Complete Personal Progress Check FRQ A for Unit 2

2.10 Finding the Derivatives of Tangent, Cotangent, Secant, and/or Cosecant Functions (Skill 1.D)

In a homework assignment, students will apply the derivative rules of Tangent, Cotangent, Secant, and Cosecant Functions. The derivatives in this homework assignment will be cumulative of all the other derivative rules but an emphasis will be placed on the new rules.

- ☐ Complete Personal Progress Checks MCQ B and FRQ B for Unit 2
- ☐ Take Unit 2 Test

Unit 3: Differentiation: Composite, Implicit, and Inverse Functions (Big Idea: Analysis of Functions)

3.1 The Chain Rule (Skill 1.C)

In a class exercise, students will work in pairs to discuss how they would solve Free-Response Question 3 – Part C from 2007 (Form B). Students will have to pay attention to the fact that they need to find the derivative $w'(t)$ when they are not given a formula that directly relates w and t; the formula given in the stem of the problem is in terms of w and r. This kind of problem should prompt a good discussion on how to find this derivative. To make things a little less complicated, I will write on the

board a pair of equations like $y = u^3$ and $u = x^2 + 1$ and ask the students to find $\frac{dy}{dx}$
I don't expect students to come up with this rule on their own, so I will demonstrate for the students on how we could write a chain rule (a chain of derivatives) to find $\frac{dy}{dx}$
Second, I will now denote $u = g(x) = x$ and ask students to find $f(g(x))$ and ask them how we could take the derivative of $f(g(x))$ and end up with our answer from before. This will now lead to the derivative of $f(g(x))$ using $f'(g(x))g'(x)$. After demonstrating a few examples, I will return to FRQ 3 and ask the students now to write a chain rule with the information they just learned. I will emphasize that students should pay attention to the units and use the units as a guide in writing a valid chain rule.

In a homework assignment, students will apply the chain rule in a variety of situations. In part one, students will be given a pair or more of equations and will have to write the chain rule using Leibniz notation to find the derivative. In part two, students will find the derivative of $f(g(x))$ without writing the chain rule. In part three, students will be given word problems like the FRQ they just worked with where they will have to write a chain rule to find the derivative paying attention to the context and units of the problem. **CR3**

3.2 Implicit Differentiation (Skill 1.E)

In a homework assignment, students will use implicit differentiation to find the derivative. In addition to performing implicit differentiation, students will need to find points along a curve where the tangent line is vertical or horizontal.

3.3 Differentiating Inverse Functions (Skill 3.G)

In a homework assignment (the day prior to this actual lesson), students will recall the properties of inverse functions. Students will be guided in steps to first find the inverse of a linear function, confirm algebraically that they obtained the correct inverse function by using $f(g(x)) = x = g(f(x))$, and confirm graphically that they obtained the correct inverse function by graphing, f, g, and $y = x$ on one set of axes and noting the symmetry of f and g about the line $y = x$. In the final part of this assignment, students will find the derivatives of f and g and be asked how their slopes compare. The next day in class, students will derive a formula for finding the derivative of inverse functions by finding the derivative of the equation $f(g(x)) = x$. Once the rule is established for finding derivatives of inverse functions, students will practice the notation for this rule using different pairs of functions. This is helpful because students often struggle with notational fluency here. **CR6**

3.4 Differentiating Inverse Trigonometric Functions (Skill 1.E)

Students will apply implicit differentiation to trigonometric inverse relations like $\sin y = x$ to generate a rule for finding the derivative of $sin^{-1}(x)$ as well as the remaining trigonometric functions.

☐ Complete Personal Progress Check FRQ B for Unit 3

3.5 Selecting Procedures for Calculating Derivatives (Skill 1.C)

In a class activity, students will work in pairs using the flowchart from ***Teaching and Assessing Module 2*** to determine which derivative rule to apply to a given function. The functions are represented as $f(x)$, y, etc. One student will show the other student how to select the derivative procedure while the other student explains why they agree or disagree with the procedure chosen. Both students then find the derivative using appropriate symbols for the derivative. They check their answers and notation. Students will switch roles after each problem. **CR6**

3.6 Calculating Higher-Order Derivatives (Skill 1.E)

In a class activity, students will work in groups of four. Each group will be given four derivative problems on index cards. For each problem, there will be three specific derivatives to find while the fourth derivative will be a general rule for finding the nth derivative of the original problem. The first three students within each group will find the indicated derivative while the fourth person will find a general rule in terms of n for finding the nth derivative. Each person in a group gets one turn at finding a rule for the nth derivative.

- ☐ Complete Personal Progress Checks MCQ and FRQ A for Unit 3
- ☐ Take Unit 3 Test

Unit 4: Contextual Applications of Differentiation and Rates of Change (Big Ideas: Change, Limits)

4.1 Interpreting the Meaning of the Derivative in Context (Skill 1.D)

In a class activity, students will start with a function $G(x)$ without context and be asked to interpret $G'(5)$. Then, context will be added to function $G(t)$ to mean the amount of unprocessed gravel arriving at a processing plant, where G is measured in tons and t is measured in hours. Since students often struggle interpreting a derivative, students will be provided with a template. The template will be: At time $t =$ _____, the function is increasing or decreasing at a rate of ___________ (units of y)/(units of x). This activity will require students to interpret different representations. In a final part of this activity, students will critique student samples from past free-response questions to learn both correct and incorrect ways of interpreting a derivative. **CR8**

4.2 Straight-Line Motion: Connecting Position, Velocity, and Acceleration (Skill 1.E)

Students will recall that average velocity is the change in position divided by the change in time, i.e., average velocity $= \frac{s(t+h)-s(t)}{h}$ and that when take $\lim_{h \to 0} \frac{s(t+h)-s(t)}{h}$ we get $s'(t)$, which refers to the instantaneous velocity at time t. Students will also recall that average acceleration is the change in velocity divided by the change in time, i.e., average acceleration is $\frac{v(t+h)-v(t)}{h}$ and when taking the $\lim_{h \to 0} \frac{v(t+h)-v(t)}{h} = v'(t) = a(t)$. Then, using guided examples, students will solve motion problems finding when the particle is at rest, reverses direction, speeding up, moves right, moves left, speeds up, and slows down using function and graphical representations. Students will have to give reasons for their answers. **CR3** **CR5** **CR6**

4.3 Rates of Change in Applied Contexts Other Than Motion (Skill 2.A)

In a homework assignment, students will find rates of change with respect to quantities other than time. Former free- response questions will be included where students will find the value of a derivative using their calculators and interpret their answers in the context of the problem. Questions will vary in their representation. Some questions will be represented in symbolic form, e.g., given $S(h)$ find $S'(4)$, while other questions will be in the form of words, e.g., "If h is the vertical distance between the graphs of functions f and g, then find the rate at which h changes with respect to x when $x = 1.8$." **CR7**

4.4 Introduction to Related Rates (Skill 1.E)

Students often struggle with Related Rates problems for numerous reasons. Therefore, before solving Related Rate problems, students will use a class activity to help them develop guidelines for solving related rate problems. The steps we will focus on during this activity consist of: 1. Drawing a picture, if applicable, to represent the problem; 2. Distinguishing between quantities that change and those quantities that don't change and labeling those quantities that change a variable; 3. Writing an equation that relates the quantities in the problem; and 4. Practice differentiating quantities in the equation with respect to time. In this step, students will need to identify the appropriate rule for differentiation based on the classification of the expression. **CR3**

4.5 Solving Related Rate Problems (Skill 3.F)

This lesson will continue from the topic 4.4 listed above. In a class activity, students will work in pairs. One student will draw a picture and label the quantities that change as variables and then pass it on to their partner for verification. Once the pair agrees on the picture and labels, they will individually write an equation that relates the quantities in the problem and switch papers to see if they agree with each other's equations. Then, the students will differentiate the equation together agreeing on their steps in finally solving the problem. **CR4**

4.6 Approximating Values of a Function Using Local Linearity and Linearization (Skill 1.F)

This class activity will begin with me passing out a sticky note and asking students to guess $\sqrt{6}$ to 3 decimal places.

The activity will continue with the steps in the handout. At the end of the activity, the student who is the closest wins a prize.

- ☐ Complete Personal Progress Check FRQ A for Unit 4

4.7 Using L' Hospital's Rule for Determining Limits of Indeterminate Forms (Skill 3.D)

If time permits, students will complete an in-class activity using their TI-Inspire to enhance their understanding of L' Hospital's Rule. As a follow up, students will complete a homework assignment using L' Hospital's Rule. The first part of the homework will require students to find the limit using both algebra and L' Hospital's Rule. Students should realize that even though both methods are capable of producing the same result, L' Hospital's Rule does have an advantage in certain problems. Limit problems in the homework will also represent the definition of derivative. In this case, the take-away is that students should realize that they could evaluate a limit representing the definition of derivative using L' Hospital's Rule.

- ☐ Complete Personal Progress Checks MCQ and FRQ B for Unit 4
- ☐ Take Unit 4 Test

Unit 5: Analytical Applications of Differentiation including Analysis of Functions (Big Idea: Analysis of Functions)

5.1 Using the Mean Value Theorem (Skill 3.E)

In a class activity, students will use the graphic organizer and template from ***Teaching and Assessing Module 3.*** The graphic organizer will help students to check for the conditions of continuity on a closed interval and differentiability on an open interval. Then, students will use the template as a strategy to write a mathematical argument. These strategies are helpful because students often have a difficult time addressing the conditions of MVT and putting all the pieces together to write an argument.

In a homework assignment, students will be expected to apply MVT in multiple representations. The problems will be represented by using functions, graphs, and tables. For each representation, students will have to explain whether or not MVT can be applied. If MVT can be applied, then students will write an argument using the template mentioned above to justify their answer. **CR5**

5.2 Extreme Value Theorem, Global Versus Local Extrema, and Critical Points (Skill 3.E)

In a class activity, students will consider several graphs of functions – continuous and discontinuous. Using the graphs, students will take away that if a function is continuous on a closed interval, then the function has both a maximum and minimum. Students will understand that a function may still have a maximum or minimum even though it is not continuous on a closed interval. **CR5**

5.3 Determining Intervals on Which a Function is Increasing or Decreasing (Skill 2.E)

In class exercises, students will start out with a sign chart to determine the intervals where a function is increasing or decreasing and then write their final answer explaining their reasoning in words. Students will have to use precise language to write their explanations. For instance, if the derivative is negative, the function is decreasing, not decreasing at a negative rate.

5.4 Using the First Derivative Test to Determine Relative (Local) Extrema (Skill 3.D)

In a homework assignment, students will use the first derivative test to determine relative extrema in various representations such as functions, tables, and graphs. Students will use a flow chart in selecting a method for finding relative extrema. The first part of the assignment will have students fill in the blanks to make a true statement. The fill in the blanks will help them develop notational fluency for justifying their answers. In part two, students will apply the flow chart for finding extrema to a variety of settings.

☐ Complete Personal Progress Check MCQ A for Unit 5

5.5 Using the Candidates Test to Determine Absolute (Global) Extrema (Skill 1.E)

In a homework assignment, students will use a graphic organizer in selecting a procedure to find absolute extrema. If EVT applies, then students will use the Closed Candidates Test. This homework assignment will include problems of various settings. Students will be expected to identify Global Extrema using functions, graphs, tables, and word problems from prior free-response questions.

5.6 Determine Concavity of Functions over Their Domains (Skill 2.E)

In a homework assignment, students will use a sign chart to determine the intervals where a function is concave up, concave down, and points of inflection. Students are expected to write complete reasons for their answers. For instance, identifying concave up as $f'' > 0$ implies f' is increasing. Students will determine concavity in various contexts using functions, graphs, and tables.

☐ Complete Personal Progress Check FRQ A for Unit 5

5.7 Using the Second Derivative Test to Determine Extrema (Skill 3.D)

In a homework assignment, students will use the second derivative test to justify extrema. Students will use the flow chart from the previous topic to help them choose a procedure for determining extrema. What to look for? Students often forget that when they are only given dy/dx and not the original function, that they need to consider the second derivative test for finding extrema and not the first derivative test. This kind of problem is addressed in topic 5.12.

5.8 Sketching Graphs of Functions and Their Derivatives (Skill 2.D)

In a class activity, students will be given graphs of several functions (polynomials, functions with corner points, and functions with vertical asymptotes) and will sketch their first derivative and in some cases second derivative.

Also, if the class is advanced, I could change this to a pair-share activity. Each pair will be given a set of four index cards. The first student will graph a function on an index card and pass the card to their partner who will sketch the derivative of that function. The second person will sketch a derivative of some function and pass the card to their partner who will sketch the function given the derivative card. Each pair will do this twice so that each student gets a turn at sketching the derivative given the function and sketching the function given the derivative. **CR4**

☐ Complete Personal Progress Check MCQ B for Unit 5

5.9 Connecting a Function, Its First Derivative, and Its Second Derivative (Skill 2.D)

In a homework assignment, students will first sketch a function by using the first derivative to determine the intervals where the graph of the function is increasing, decreasing, and relative extrema, in addition to, using the second derivative to determine the intervals where the graph of the function is concave up, concave down, and points of inflection and justify their answers for each. Secondly, students will use their graphing calculator to confirm that their graph is correct. Third, once students verify their graph is correct, they will use their graph of the function to graph the first derivative. Next, students will graph the second derivative from the graph of the first derivative. Students will align all three graphs under each other, i.e., f, f', and f''. Students will then draw vertical lines connecting the following relationships: horizontal tangents on the graph of f is where the first derivative is zero, the relative extrema on the graph of f are points where the graph of the first derivative changes signs, the points of inflection on the graph of f line up with the relative extrema on the graph of f' which supports where the graph of f'' changes signs.

5.10 Introduction to Optimization Problems (Skill 2.A)

Students will complete a class activity in small groups. Each group will be given one sheet of 8.5 by 11 inch paper. The group has to decide what size square to remove from each corner to produce the greatest volume. The group will remove that size square from each corner, fold up all the sides, and tape the edges to form a rectangular box. This activity is very hands-on and will help students make the transfer to more challenging optimization problems as far as what is the ultimate goal of solving optimization problems. The next day in class, students will learn how to use calculus to arrive at the answer and the group with the size square closest to the actual answer wins a prize.

5.11 Solving Optimization Problems (Skill 3.F)

In a class activity, students will learn a 5-step process for solving optimization problems. Students will receive a set of cards consisting of word problems, pictures, and various equations. The students will separate the cards into three groups. Each group will require a problem statement, a diagram, an optimization equation, a constraint equation, and an optimization function. Then, the students will complete the problems using the steps listed in the chart. Something to watch for is that many students fail to justify their answers using information about the derivative. If students don't know the closed interval they are working on, then EVT cannot be applied; hence, they should try using a global argument for justifying their absolute maximum or absolute minimum.

5.12 Exploring Behaviors of Implicit Relations (Skills 1.E and 3.E)

This topic will be covered back in topics 5.5 and 5.7. In a homework assignment, students will be given problems where they will have to find absolute extrema given an equation with dy/dx.

- ☐ Complete Personal Progress Checks MCQ C and FRQ B for Unit 5
- ☐ Take Unit 5 Test

Unit 6: Integration and Accumulation of Change (Big Ideas: Change, Limits, Analysis of Functions)

6.1 Exploring Accumulations of Change (Skill 4.B)

Class will start with a velocity vs. time graph, where $v(t) = 70$ miles per hour from $t = 0$ to $t = 2$ hours.

From that information, students will easily know that the total distance traveled is 140 miles. What is important here is for the students to understand that 140 corresponds to the area under the graph of $v(t) = 70$ from $t = 0$ to $t = 2$ hours. I will also reinforce that as time increases from 0 to 2, that the area under the graph is accumulating more distance because $v(t)$ remains positive. It is also not surprising that the unit belonging to the answer of 140 is miles because when multiplying the units of $v(t)$ and t you get miles. (Note: These are also the units belonging to the dimensions of the rectangle.) I emphasize to my students that we started with velocity measured in mph and ended up with distance measured in miles, so one might say that we are using an inverse process to differentiation. This is where I will show my students how to represent this scenario as a definite integral. Once I establish this problem using definite integral notation, my story continues with $v(t) = -70$ mph from $t = 2$ to $t = 4$ hours. Once students understand that whenever the area of a region bounded by $v(t)$ and the t-axis lies beneath the t-axis, the area represents a negative change in position over the given time, I will ask my students to evaluate many different integrals such as $\int_0^4 v(t)\,dt$, $\int_0^4 |v(t)|\,dt$, using geometry/area and interpret their answer in the context of the problem. In this short practical problem, I am able to build notational fluency with the definite integral, determine the units belonging to the answer of the definite integral, interpret the definite integral in the context of the problem using units, and understand the definite integral as an accumulation function.

6.2 Approximating Areas with Riemann Sums (Skill 1.F)

In a homework assignment, students will approximate the definite integral using Left, Right, and Midpoint Riemann Sums. Problems will be broken into parts consisting of functions, tables, and graphs.

6.3 Riemann sums, Summation Notation, and Definite Integral Notation (Skill 2.C)

In a class exercise, students will learn how to use the definition of the definite integral to find the exact value of a definite integral. Class will begin with a discussion about how we could improve our approximations from last night's homework in topic 6.2. Hopefully, students will say by increasing the number of rectangles. I will continue this discussion by asking them how many rectangles should we increase it to, and hopefully they will say an infinite number of rectangles. Then, I will use technology (***Calculus In Motion*** by Audrey Weeks) that will show by increasing the number of intervals, the limit of the sum of the areas of the rectangles approaches a finite number.

Once the definition of the definite integral is established, students will complete a homework assignment finding the exact value of the definite integral using the definition. Then, they will have to write a definite integral given the limit expression representing the definition of the definite integral.

☐ Complete Personal Progress Check FRQs B for Unit 6

6.4 The Fundamental Theorem of Calculus and Accumulation Functions (Skill 1.D)

For a homework assignment, students will complete the activity on Integral-Defined Functions. The activity indicates three different integral-defined functions using the graph of $f(t)$. Students will use geometry/area to find each value in the table. Then, students will graph the values for each function in the table on the grid provided. The students will take away that, although the three functions have a different lower limit as a constant, their rates of change are all the same and that the derivative of each of the integral-defined functions is the graph of f (regardless of the lower limit). Another take-away from the graph is that functions can have the same rate of change yet differ by a constant.

6.5 Interpreting the Behavior of Accumulation Functions Involving Area (Skill 2.D)

As a follow-up to topic 6.4, students will complete an activity in class applying the Fundamental Theorem of Calculus to understanding the behavior of an integral-defined function. If $g(x) = \int_a^x f(t)dt$, then students will use the connections $g' = f$ and $g'' = f'$ to answer where the function g is increasing, decreasing, concave up, concave down, as well as relative extrema and points of inflection. The vocabulary students used back in Unit 5 to describe the behavior of a function is the same vocabulary they will use to describe the behavior of an integral-defined function.

6.6 Applying Properties of Definite Integrals (Skill 3.D)

Students will complete a homework assignment applying Properties of Definite Integrals. This assignment will consist of various parts. In part one, students will be given a graph of a function f and a definite integral value for a function g, and constant function Students will find the value of the given definite integral such as $\int_1^3 (2f(x)+3g(x)-h(x))dx$ using definite integral properties such as $\int_a^b cf(x)dx = c\int_a^b f(x)dx$, $\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$,

$\int_a^b f(x) + \int_b^c f(x)dx = \int_a^c f(x)dx$, and $\int_a^b f(x)dx = -\int_b^a f(x)dx$. Students will have to use area to find the value of the definite integral for f. In part two, students will be given multiple-choice questions from prior exams or the item bank where students have to apply these properties. **CR6**

☐ Complete Personal Progress Checks MCQ A for Unit 6

6.7 The Fundamental Theorem of Calculus and Definite Integrals (Skill 3.D)

In a homework assignment, students will use both parts of the Fundamental Theorem of Calculus (FTC) to solve problems. In part one, students will complete the statement using either part of the Fundamental Theorem of Calculus. These are great problems to help students establish appropriate notation in applying the FTC. Students often have difficulty connecting the FTC to problems when the notation varies. In part two, students will use the evaluation part of the FTC to find missing function values; some problems will require technology. In part three, students will be given a graph of a function and will have to use area using either part of FTC.

6.8 Finding Antiderivatives and Indefinite Integrals: Basic Rules and Notation (Skill 4.C)

The class activity will start by having students come up with a position function for an object whose velocity is given by $v(t) = 3t$. This class discussion will generate a rule (general power rule) for finding antiderivatives of the form $\int u^n du$, where $n \neq -1$. Students will often have to perform algebra first before applying this rule. **CR6**

6.9 Integrating Using Substitution (Skill 1.E)

Students will be given an indefinite integral where they have to recognize the integrand involves the chain rule. A class discussion will include how we could come up with a function whose derivative is in the integrand. This discussion will lead to integration by u-substitution. To help students, they will learn a 3-step process. Students will use the two activities from the ***Teaching and Assessing Modules*** to help them with sequencing the steps in the u-substitution process and error analysis. For each indefinite rule the students learn, the following day they will apply the FTC to evaluating the definite integral using that new rule. Students will learn to use the definite integral feature on their calculator to verify their answer to the definite integral. **CR7**

☐ Complete Personal Progress Check FRQs A for Unit 6

6.10 Integrating Functions using Long Division and Completing the Square (Skill 1.E)

In a homework assignment, students will first have to use Long Division or Completing the Square before finding the antiderivative.

Topics 6.11, 6.12, and 6.13 are BC-only.

6.14 Selecting Techniques for Antidifferentiation (Skill 1.C)

In an activity similar to how the students had to select procedures for derivatives, they will also learn to select techniques for antidifferentiation. In a class activity, students will use the strategy of error analysis. Written on the white board around the room will be six integration problems broken into stations. Students will visit each station in pairs, discuss what the error is in the integration process, and then redo the problem correcting the error and agreeing upon their solutions.

☐ Complete Personal Progress Check MCQ B for Unit 6

☐ Take Unit 6 Test

Unit 7: Differential Equations (Big Idea: Analysis of Functions)

7.1 Modeling Situations with Differential Equations (Skill 2.C)

In a homework assignment, students will write a differential equation to model a given situation. Prior to the homework, a variety of examples will be demonstrated in class using a chart. In the chart, verbal statements will be associated with mathematical notation; e.g., "the rate of change of y with respect to t is proportional to the amount of y" would be associated with, "$\frac{dy}{dt} = Ky$." Once students know the proper notation, writing a differential equation to model a situation becomes easier for them.

7.2 Verifying Solutions to Differential Equations (Skill 3.G)

In a homework assignment, students will find first or second derivatives to verify solutions to differential equations. **CR4**

7.3 Sketching Using Slope Fields (Skill 2.C)

7.4 Reasoning Using Slope Fields (Skill 4.D)

Topic 7.5 is BC-only.

7.6 Finding General Solutions Using Separation of Variables (Skill 1.E)

7.7 Finding Particular Solutions Using Initial conditions of Separation of Variables (Skill 1.E)

I put 7.3, 7.4, 7.6, and 7.7 together in one lesson, revisiting the same topics in different contexts for a few days. In a class activity, I will start with students sketching a slope field for a differential equation in terms of just x. By having them sketch the slope field first, they get a feel for what is meant by a slope field while learning how to draw directed line segments that represent the slope of the curve at that point. Also, by starting with a differential equation of just x, students will notice that all the directed line segments are the same vertically, and that if the differential equation was just in terms of y, then all the directed line segments would be the same horizontally. After students construct a slope field, I will give them several points through which they will draw particular solutions and talk about the family of functions that are represented in the slope field. Students start to reason that if the differential equation just had an x then the family of curves (the general solutions) would be represented by parabolas or if the differential equation just had an x^2, then the family of curves (the general solutions) would be represented by the shape of x^3. By starting with these basic differential equations, students are able to make connections from the differential equation → to its slope field → to its general solution → and then finally to its particular solution.

The next type of differential equation my students will work with involves x and y. They will do a matching activity between the slope field and the differential equation. This is where students will have to increase their reasoning with slope fields because they will be looking for where the slope does not exist (which implies vertical tangents), where the slope is zero (which implies horizontal tangents), and where the slopes are positive or negative. Then, we will construct slope fields given a differential equation in terms of x and y and find its particular solution through a given point. A particular example I consider every year is the differential equation given by $\frac{dy}{dt} = -\frac{x}{y}$. Usually students have no issue with the constructing the slope field, but they make an error when they draw the particular solution through the point $(0, 2)$ $(0, 2)$ because they make a full circle. Students need to be reminded about particular solutions to differential equations. Particular solutions are functions (that are unique) and are continuous under appropriate conditions. Therefore, the graph of this particular solution belonging to the differential equation above is a semicircle (a function of x) of radius 2 with holes on the x – axis at $(-2, 0)$ and

(2, 0). The domain belonging to the particular solution equation must also satisfy the differential equation. The domain for the particular solution, $y=\sqrt{4-x^2}$ satisfying the point (0, 2) and $\frac{dy}{dt}=-\frac{x}{y}$ is (-2, 2).

As a follow-up to these types of problems, students will be assigned released free-response questions from the College Board website.

☐ Complete Personal Progress Check FRQs A and B for Unit 7

7.8 Exponential Models with Differential Equations (Skill 3.G)

As an extension to topics 7.1 through 7.7, students will not only write a differential equation based on the written problem, but will have to solve the differential equation using the initial condition. Some solutions to differential equations will be modeled by exponential functions while others will not. Students often struggle with solving differential equations when there is context. Therefore, to help students transfer what they have learned so far to solving free-response questions like 2011 problem 5, I have created a worksheet for homework that will scaffold the skills necessary to solve such problems. The homework will start by having students practice separating the variables, which is essential for students in order to solve the differential equation. Then, students will find the particular solutions to differential equations in terms of variables other than x and y. (Students need practice with other variables.) Finally, students will solve the differential equation modeling a particular solution. Some of these problems will be former free-response questions.

Topic 7.9 is BC-only.

☐ Complete Personal Progress Check MCQ for Unit 7

☐ Take Unit 7 Test

Unit 8: Application of Integration

8.1 Finding the Average Value of a Function on an Interval (Skill 1.E)

In a classroom discussion, I will show the students how to derive the formula for finding the average value of a function on a closed interval. Students will be assigned a homework assignment where they will have to find the average value of a function in multiple representations including functions, graphs, and written contexts. **CR4**

8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals (Skill 1.D)

In a homework assignment, students will apply the Fundamental Theorem of Calculus to problems pertaining to position, velocity, and acceleration. In some problems, students will be given information about the velocity of an object in multiple representations and will answer questions about the object's position. In other problems, students will be given information about the acceleration of an object, the object's velocity and position at a time t, and will be asked to find particular solutions for the object's velocity and position.

8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts (Skill 3.D)

This can be a challenging topic for my students because they have not internalized some concepts nor linked them to the appropriate notation. For example, some questions are asking for an average rate of change while other questions are asking for the average value. Because these questions seem very similar to each other, students are not sure what notation to write. These types of questions, usually referred to as "Rate Questions," involve rate of change, average rate of change, average value, find the amount of a quantity given the rates of change, etc. In a class activity, I will use a flowchart to help my students learn how to set up the integral. In the activity, students will write what functions they are given in the middle of the flowchart and label the units. The units and notation mean everything and cannot be ignored. Working from the middle of the flowchart, students will work upwards, interpreting various integral expressions using units and context; while working downwards, students will interpret derivatives using units and context. Some problems will require the graphing calculator for both derivatives and integrals. **CR7**

- ☐ Complete Personal Progress Check FRQ A for Unit 8

Topics 8.4 through 8.6 Area Between Curves (Topic 8.4, Skill 1.E; Topic 8.5, Skill 1.E; Topic 8.6, Skill 2.B)

Students will complete a homework assignment where they will find the area between two or more curves. Throughout these topics, students will have to find areas between functions of x and as well as areas between functions of y. Some problems will have more than two points of intersection Some problems will require the students to use their graphing calculator's equation solve to find the points of intersection, then use the calculator to graph the functions to determine which graph is higher, and finally use the calculator's numerical integration feature to approximate the area. **CR7**

- ☐ Complete Personal Progress Check MCQ A for Unit 8

Topic 8.7 and Topic 8.8 Volumes with Cross Sections (Topic 8.7, Skill 3.D; Topic 8.8 Skill 3.D)

Students will complete a homework assignment where the base of a solid is the region enclosed by given equations and the known cross sections consist of either a square, rectangle, triangle, or semicircle.

To help students visualize what these solids look like, I will use the software – Calculus In Motion.

Topics 8.9 through 8.12 (Topic 8.9, Skill 3.D; Topic 8.10, Skill 2.D; Topic 8.11, Skill 4.E; Topic 8.12, Skill 2.D)

In a class activity, students will find volumes of revolutions by using the washer and disk methods. I will start the lesson with an enclosed region bounded by the graph of $y = \sqrt{x}$ and the x – axis from $x = 0$ to $x = 4$. Next, using Calculus In Motion, the software will revolve one rectangle about the x – axis showing the result of one cross section – a disk. Using the volume formula for a cylinder, I will derive with the class a formula for finding the volume of revolution by disk. I will reinforce to students how important it is to draw the rectangle in the enclosed region. The height of the rectangle represents the length of the radius and when the rectangle is drawn vertically, the length of the rectangle is the top function minus the bottom function resulting in an integrand in terms of x.; and when the rectangle is drawn horizontally, the length of the rectangle is the right function minus the left function where integrand is in terms of y. After a few examples and a homework assignment, I will return to this same problem (same region bounded by the x – axis) the next day where the axis of rotation gets dropped to the line $y = -2$. Using Calculus In Motion, I will revolve one rectangle about the line $y = -2$, and the students will see that the result of one cross section is a washer. Using a class discussion, I will derive with the students a formula for finding the volume by the washer method. To help students with setting up the formula, I will require students to draw the rectangle perpendicular to the axis of rotation; this way, the students will be able to recognize the big radius (representing the radius of the outer cylinder) and the clear radius (representing the radius of the inner cylinder). Volume by washer is recognized in my class by the formula

$$V_W = V_{big\ radius^2} - V_{clear\ radius^2}$$

Once students learn all the volumes of revolution methods, they often get confused on how to set up the integral. To help my students learn the set-up of volumes by disk and washer, we will complete a chart of problems that allows them to make the connection between the picture (the enclosed region and axis of rotation) and the appropriate volume method.

After Topic 8.9, students may complete Personal Progress Check FRQ B for Unit 8.

Topic 8.13 is BC-only.

- ☐ Complete Personal Progress Check MCQ B for Unit 8
- ☐ Take Unit 8 Test

Application of Calculus Project:

Students will complete a project pertaining to optimization. Students will come up with a product that they want to manufacture and decide on how much of that product they want to package and sell. Students will provide in detail how much it will cost for them to manufacture and package their product. Then, students will use calculus to determine the dimensions of their packaging that will determine the least amount of material used to package their product. Finally, students will use all of their information to determine how much they should sell their product for to make their desirable profit. **CR8**