2021 Digital Exam Practice

AP Calculus AB

Scoring Materials for Digital Exam Practice

Please note: the digital exam practice resource was developed for students to complete technology checks, experience the digital platform, and practice answering exam questions, including each type of multiple-choice and free-response question they will encounter on exam day. Be aware that on exam day students will complete six free-response questions while this practice includes only three.

This digital exam practice is not a full-length exam, and it does not represent the complete scope of content and skills that students will see on the actual AP exam. This digital exam practice includes only content that would typically be taught in the first half of the school year, following the unit sequence in the AP Calculus AB Course and Exam Description. For more information on the 2021 Exam format, please visit: apcentral.collegeboard.org/pdf/ap-2021-exam-formats.pdf

AP Exams are scored differently than traditional high school or college exams. When an AP Exam is administered, psychometric analysis determines the score ranges corresponding with each AP Exam score (5, 4, 3, 2, and 1) based on a composite score scale that combines and weights the different exam parts. Earning 40-50% of the available points can result in a score of 3 or better on many AP Exams. However, because the number of points corresponding with each AP Exam score can vary on different exams, students and teachers should not use the results of the digital exam practice to predict performance on the 2021 AP Exam.

Multiple-Choice Answer Key

Section 1.A Calculator not required

Multiple-Choice Question	Answer
1	D
2	D
3	D
4	A
5	A
6	A
7	D
8	D
9	A
10	A

Section 1.B Calculator required

Multiple-Choice Question	Answer
11	C
12	B
13	D
14	B

Part A (AB): Graphing calculator required
Question 1 — 9 points

General Scoring Notes

The model solution is presented as a student would provide in a typed response.

Because responses are typed, minor typographical errors and/or omissions do not necessarily disqualify a response from earning a particular point, provided the response is otherwise correct and clear.

Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\frac{t^2}{18}$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

a. Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

Model Solution	Scoring	
G'(5) = –24.588 (or –24.587)	$G'(5)$	**1 point**
The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time t=5 hours.	Interpretation and units	**1 point**

Scoring notes:

- The interpretation must include the correct direction (decreasing), the correct units (tons per hour per hour) and t=5.
- A response indicating that the rate at which gravel is arriving is decreasing at a rate of -24.588 tons per hour per hour at time t=5 hours does not earn the second point.

Total for part a — 2 points

b. Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

G(5)=98.140764 < 100	Compares $G(5)$ to 100	**1 point**
At time t=5, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at t=5.	Conclusion	**1 point**

Scoring note:

- An argument based on the sign of $G(5) - 100$ is also acceptable.

Total for part b 2 points

c. By weight, the plant processes gravel at a rate of 100 tons per hour and a ton of gravel occupies a volume of 19 cubic feet.

Processed gravel is poured into a conical pile such that the ratio of the height to the radius is 2 to 3.

The volume of a cone, V, is given by $V = \frac{1}{3}\pi r^2 h$.

Identify by label (P, Q, R, S, or T) which of the following equations would be appropriate to use to find the rate of change, with respect to time, of the radius of the conical pile for a given radius. Then, find the rate of change, with respect to time, of the radius of the conical pile when the radius is 10 feet. Be sure to provide all values used in your computation.

P. $\frac{dV}{dt} = \frac{1}{3}(2\pi rh + \pi r^2)\frac{dr}{dt}$

Q. $\frac{dV}{dt} = \frac{2}{3}\pi r^2 \frac{dr}{dt}$

R. $\frac{dV}{dt} = \frac{3}{2}\pi r^2 \frac{dr}{dt}$

S. $\int_0^8 dV = \frac{2\pi}{3}\int_0^8 r^2 dr$

T. $\int_0^8 dV = \frac{3\pi}{2}\int_0^8 h dh$

Since h=(2/3)r, V=(2pi/9)r^3. Equation Q would be appropriate to use to find dr/dt.	Equation Simply writing "Q" earns the point.	**1 point**
dV/dt = (100 tons/hour)(19 cubic feet/ton) dV/dt = 1900 cubic feet per hour and $r = 10$ feet.	$\frac{dV}{dt} = 1900$	**1 point**
dr/dt = 9.072 (or 9.071) feet per hour	$\frac{dr}{dt}$	**1 point**

Scoring note:

- The second point is for demonstrating that dV/dt=1900. That may be demonstrated by an explicit statement, as in the model solution, or by using 1900 for dV/dt in an equation. For example, the second point is likewise earned for "1900=(2pi/3)(10^2)dr/dt ," or similar, even if there are minor typographical errors in the righthand side of the equation.
- A response that incorrectly uses 100 tons/hour for dV/dt will not earn the second point but is eligible to earn the third point for the consistent answer of dr/dt = 3/(2π), or 0.4775.
- Errors in solving for dr/dt will result in not earning the third point.

Total for part c 3 points

d. Let $\boldsymbol{t = H}$ hours be the time during the workday ($\boldsymbol{0 \le t \le 8}$) when $\boldsymbol{G(t) = 100}$.
$\boldsymbol{N(t) = 20(t - H)\sin\left(\frac{t}{27}\right)}$.
Find $\boldsymbol{\lim_{t \to H}\left(\frac{G(t)-100}{N(t)}\right)}$ or state that it does not exist. Justify your answer.

For this question, you can type "LIM" to indicate the limit as $\boldsymbol{t}$ approaches H, even though this representation does not specify that $\boldsymbol{t}$ approaches H.

For example, LIM((G(t)-100)/N(T)) will be interpreted to mean $\boldsymbol{\lim_{t \to H}\left(\frac{G(t)-100}{N(t)}\right)}$.

G(t)=100 at t=4.923480296, so H=4.923. Both G and N are continuous for all t. Since LIM(G(t)-100) = G(H)-100 = 0 and LIM(N(t)) = N(H) = 0, LIM((G(t)-100)/N(T)) = LIM(G'(t)/N'(t))	Applies L'Hospital's rule	**1 point**
G'(H)/N'(H) = -6.618 (or -6.617)	Answer	**1 point**

Scoring notes:

- Since *G* and *N* are familiar functions, a response that does not state that they are continuous is eligible to earn both points.
- A response that does not explicitly state a value for H is eligible to earn both points.
- To earn the point for applying L'Hospital's Rule, the response must confirm that L'Hospital's rule applies and present an expression involving a limit of the ratio of the derivatives of G and H.
- LIM G'(t)/N'(t) will be interpreted to mean LIM(G'(t)/N'(t)).
- Errors in evaluating the expression, including the results of using incorrect values or rounding for H, will be assessed in the third point.

Total for part d **2 points**

Total for question 1 **9 points**

Part A (AB/BC): Graphing calculator required
Question 2 — 9 points

General Scoring Notes

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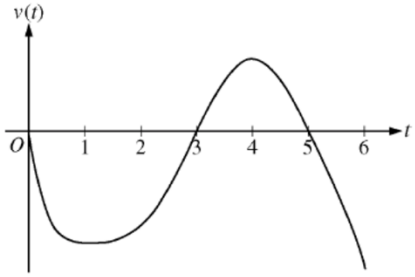
Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.



Graph of v

A particle, P1, moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function V1, whose graph is shown. The velocity of P1 is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The position, X1, of P1 at $t = 3$ is 8.

a.

Model Solution	Scoring	
During what time interval(s), if any, on the interval $0 \le t \le 6$ is the acceleration of P1 negative? Give a reason for your answer.		
The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$, since velocity is decreasing on those intervals.	Answer (intervals)	**1 point**
	Reason	**1 point**

Scoring note:

- Responses should reason directly from the given graph of velocity, as in the model solution.

Total for part a — 2 points

b. On the interval $2 < t < 3$, is the speed of P1 increasing or decreasing? Give a reason for your answer.

The speed is decreasing on the interval $2 < t < 3$, since on the interval, $v < 0$ and v is increasing.	Answer with reason	**1 point**

Scoring note:

- The response must address both the sign of v and that it is increasing.

Total for part b **1 point**

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c. A second particle, P2, also moves along the x-axis. For $0 \le t \le 6$, the position of P2 at time, t, is given by $X2(t) = (2.7t)\sin(1.25t + 2.9) - 0.9$.

Identify by label (Q, R, S, T, or U) which of the following is a correct expression for $V2(t)$, the velocity of P2 at time, t.

Then, determine whether P2 is moving to the left, right, or not at all at $t = 3$ and explain your reasoning.

Q. $\int_0^t ((2.7x)\sin(1.25x + 2.9) - 0.9)dx$

R. $\int_0^t (2.7x)\sin(1.25x + 2.9)dx - 0.9$

S. $2.7(1.25\cos(1.25t + 2.9))$

T. $2.7t(1.25\cos(1.25t + 2.9) - 2.7\sin(1.25t + 2.9))$

U. $2.7\sin(1.25t + 2.9) + 2.7t(1.25\cos(1.25t + 2.9))$

Expression U gives V2(t).	Expression	**1 point**
V2(3)=10.4197642>0	Considers sign of $V2(3)$	**1 point**
Since V2(3) > 0, P2 is moving to the right at $t = 3$.	Answer with reason	**1 point**

Scoring note:

- A response that identifies Q or R for $V2(t)$ is ineligible for any points on part c.
- A response that identifies S or T for $V2(t)$ is eligible only for the second point.
- A response that identifies the expression U and presents an incorrect value for V2(3) is eligible for the second point, but not for the third.

Total for part c **3 points**

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d. Determine whether P2 is moving towards or away from P1 at $t = 3$. Explain your reasoning.

Since V1(3)=0, P1 is moving neither to the right nor to the left at t=3, while P2 is moving to the right at $t = 3$.	Compares directions of motion of P1 and P2 at $t = 3$	**1 point**
Since X1(3)=8 and X2(3)=2.005, P2 is to the left of P1 at t=3.	Considers relative positions of P1 and P2 at $t = 3$	**1 point**
Since P2 is to the left of P1 and moving to the right, while P1 is not moving to the left or to the right, P2 is moving towards P1.	Answer with explanation.	**1 point**

Scoring note:

- A response importing an incorrect value for V2(3) from part c is eligible to earn the first point for a consistent comparison of directions of motion of P1 and P2 at t=3.
- Presenting an incorrect value for X2(3) will be assessed in the third point.

Total for part d 3 points

Total for question 2 9 points

Part B (AB or BC): Graphing calculator not required
Question 3 **9 points**

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Answers (numeric or algebraic) need not be simplified. Answers given as a decimal approximation should be correct to three places after the decimal point. Within each individual free-response question, at most one point is not earned for inappropriate rounding.

t (minutes)	0	1	2	3	4	5	6
$V(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function V, where t is measured in minutes. Selected values of $V(t)$, measured in ounces, are given in the table.

a.

Model Solution	Scoring	
Use the data in the table to approximate $V'(3.5)$. Show the computations that lead to your answer and indicate units of measure.		
V'(3.5) ~ (f(4)-f(3))/(4-3) = (12.8-11.2)/1 = 1.6 ounces per minute	Difference quotient and approximation (simplification of numeric value is not required)	**1 point**
	Units	**1 point**

Scoring note:

- (12.8 – 11.2)/1 shows a difference quotient and (unsimplified) numeric value for the approximation and is sufficient to earn the first point

Total for part a **2 points**

b. The amount of coffee in the cup, in ounces, is also modeled by $\boldsymbol{W(t) = 16 - 16e^{-0.4t}}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $\mathbf{t = 5}$. Show the work that leads to your answer.

W'(t) = –16(–0.4)e^(-0.4t) = 6.4e^(-0.4t) W'(5) = 6.4 e^[(-0.4)(5)] = 6.4e^(-2) = 6.4/e^2 ounces per minute	$W'(t)$ (or W'(5)) is of the form $ke^{-0.4t}$, where $k = 6.4$ (correct) or $k = -16$ (ineligible for the second point).	**1 point**
	$W'(5)$, provided the first point is earned and $k = 6.4$ (simplification of numeric value is not required)	**1 point**

Scoring notes:

- -16 e^[(-0.4)(5)] presents a chain rule error. This response would earn the first point for differentiating the constant, a difference, and e^u, but is not eligible to earn the second point.
- Potentially ambiguous use of parentheses, such as e^(-0.4)(5), will be interpreted to mean the correct response, unless the response goes on to commit to an incorrect answer, as in 5 e^(-0.4).
- A calculator is not required for this question and we do not expect responses to present a decimal approximation for this question. If they do, the response should be 0.866.
- An unsupported answer of W'(5)=0.866 earns no points, because it does not present the form of the derivative and is ineligible for the answer point.
- Units are not required to earn either point in part b.

Total for part b **2 points**

c. Evaluate $L = \lim_{t \to \infty}(16 - 16e^{-0.4t})$ and interpret the meaning of L in the context of the problem.

L=16	Value of the limit	**1 point**
L gives the end behavior for W(t), which models the amount of coffee in the cup at time t. L=16 means that as time grows without bound, the amount of coffee in the cup approaches 16 ounces.	Interpretation in context	**1 point**

Scoring notes:

- Interpretation must address both time and the amount of coffee in the cup in ounces.

Total for part c **2 points**

d. For $1 \le t \le 6$, the cost, in dollars per ounce of coffee at time, t, can be modeled by $M(t) = \frac{4}{V(t)}$. It is known that $M'(5) = -0.02$.

First, interpret the meaning of the value of $M'(5)$ in the context of the problem. Include consideration of units.

Then, identify by label (P, Q, R, S, or T) which of the following equations can be used to find $V'(5)$. Identify all values you would substitute into that equation in order to solve for $V'(5)$.

P. $M'(5) = \frac{(V(5))\cdot 0 - 4V'(5)}{(V(5))^2}$

Q. $M'(5) = \frac{(V(5))\cdot 0 + 4V'(5)}{(V(5))^2}$

R. $M'(5) = \frac{4V'(5) - (V(5))\cdot 0}{(V(5))^2}$

S. $M'(5) = \frac{-4}{(V(5))^2}$

T. $M'(5) = \frac{0}{V'(5)}$

M'(5) = –0.02 means that at t=5 minutes, the cost per ounce of coffee is decreasing at a rate of 0.02 dollars per ounce per minute.	Interpretation and units	**1 point**
Equation P can be used to find V'(5). M'(5) = –0.02 and V(5) = 13.8 OR: –0.02 = -4V'(5)/(13.8^2)	Identification of equation	**1 point**
	Identification of values for M'(5) and V(5)	**1 point**

Scoring notes:

- Interpretation must address the direction (decreasing), the time (t=5 minutes) and the units. A response that states, "decreasing at a rate of -0.02 …" will not earn the first point.
- If a response identifies Q or R, the response will not earn the second point, but is eligible for the third.
- If a response identifies S or T, the response will earn neither the second nor the third point.
- Suppose a response presents only a correct form of the equation: –0.02 = -4V'(5)/(13.8^2), for example. Then the response will earn the second point for the equation and the third point for values for M'(5) and V(5), which are evident in the equation.

Total for part d 3 points

Total for question 3 9 points