Commentary on the Instructions for the Free-Response Section of the AP® Calculus Exams (revised January 2011)
by the AP Calculus Development Committee & Chief Reader

This report includes the most recent version of the general instructions for the free-response section, which will be in effect for the 2011 exams. This document presents some explanations that will help AP students and teachers interpret these instructions.

AP teachers are encouraged to review these free-response instructions with their students well in advance of the exams. By design, the instructions are kept as concise as possible. In a classroom setting, the teacher has the opportunity to discuss, elaborate on, and to emphasize key points. A thorough understanding of the general instructions will certainly improve a student's ability to succeed on the exam. Generally, the instructions simply stress the importance of clearly communicating written mathematical work. The anonymous scorers (known as Readers) of the exams are interested readers of the students' work and recognize and reward complete, coherent arguments and explanations.

Each bulleted instruction is presented as it appears on the exams, followed by additional comments *in italics* from the Development Committee. Some of these comments are in the form of answers to frequently asked questions (FAQs). Where appropriate, examples from recent scoring guidelines are cited to provide additional illustrations.

INSTRUCTIONS FOR SECTION II

The questions for Part A are printed in the green insert and the questions for Part B are printed in the blue insert. You may use the inserts to organize your answers and for scratch work, but you must write your answers in the pink Section II booklet. No credit will be given for work written in the inserts. Write your solution to each part of each question in the space provided for that part in the Section II booklet. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may keep the green insert and continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

Comment: *If a student wants the Reader to ignore some erroneous work, crossing it out is not only faster, but also a clearer indicator than erasing. Students may use pencil or pen (black or dark blue ink) for the free-response section.*

FAQ: *What happens if a student provides two different solutions for the same question?*

Answer: *If it is not clear that the student has abandoned one solution attempt for another, Readers are usually instructed to score <u>both</u> solutions, find the arithmetic mean of the two scores, and round the* *resulting mean down to the nearest whole number. For example, if the two solutions are scored 1 and 4 points, respectively, the student is awarded 2 points (the mean, 2.5, is rounded down).*

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.

Comment: *The instruction "Show all of your work" is important throughout Section II of the exams.*

Comment: *In questions where two or more functions are under discussion (for example, a function and one or more of its derivatives), it is very important for students to make unambiguous references in both their labeling and their prose. In a written explanation, ambiguous references to "the function" or "it" are difficult to interpret when there is more than one "thing" to which these could refer. In general, Readers do not infer a specific reference when there is more than one possibility.*

FAQ: *What happens if a student provides a perfectly correct final answer with no supporting work?*

Answer: *Questions on the free-response section of the exam usually require supporting work to obtain an answer. On this section of the exam, students are expected to show the reasoning and methods that lead to an answer. Therefore, an isolated or separate answer without any supporting reasoning or computations usually earns no credit. In addition, an incorrect answer without supporting work will not earn any partial credit.*

Comment: *When instructed to justify an answer, students are expected to provide an explanation of the mathematical basis for their results or conclusions. For example, to justify the location of a relative extremum of a function, a student could invoke the First or Second Derivative Test accompanied by evidence that the hypotheses are satisfied. In other cases, a student could show that the hypotheses are satisfied for the relevant theorem, such as the Intermediate Value Theorem or the Mean Value Theorem. Statements of the form "from my calculator I can see that ..." are not sufficient.*

Comment: *In some questions students may be specifically reminded to include evidence of the reasoning, strategies, or computations they used to arrive at their answers. The phrasing of a reminder can take several forms, as illustrated below. The reminders add emphasis to the instruction "show all your work." The absence of such a reminder does not imply students may ignore the instruction to show all work. The underlying message is the same for all questions: <u>answers usually need supporting work to qualify for credit</u>.*

"Justify your answer."

As outlined above, this instruction requires a mathematical argument to support the claim or conclusion. This is where the application (often with citation) of a theorem, property, or test is generally needed.

Examples: *The existence of a value satisfying a given condition might be justified by showing that the hypotheses are satisfied for the relevant theorem, such as the Intermediate Value Theorem or Mean Value Theorem, or for a test such as the Second Derivative Test. See the scoring guidelines for 2010 AB2/BC2, 2010 AB3, 2009 AB2/BC2, 2009 AB6, 2009 Form B AB3/BC3, 2008 AB3, 2008 AB4/BC4, 2007 AB6 for examples.*

FAQ: *Are sign charts acceptable in justifying either a local or an absolute extremum of a function?*

Answer: *A sign chart is an annotated number line that relates the graphical behavior (increasing/decreasing, concave up/down) of one function with the sign behavior (positive/negative) of another. Sign charts are a useful tool to investigate and summarize the behavior of a function. However, sign charts, by themselves, will not be accepted as a sufficient response when a question asks for a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. For more details on this topic, consult "On the Role of Sign Charts in AP Calculus Exams for Justifying Local or Absolute Extrema," which is available on the Calculus AB and Calculus BC Home Pages at AP Central®.*

"Give a reason for your answer."
"Explain your reasoning."
"Explain your answer."

*These reminders ask for a basic mathematical connection to support the answer to a "This or that?" kind of question, especially questions that ask for an interpretation. Another version of "Give a reason for your answer" that might follow a "Yes or no?" question is "Why or why not?" Here are some of the forms of questions that are often followed by such a reminder: Is *blank* increasing or decreasing? Is *blank* concave up or concave down? Is *blank* the location of a local maximum, local minimum, or neither? OR: Does *blank* happen? How many times does *blank* happen?, etc.*

Examples: *The mathematical (calculus) connection given as a reason might make some reference to the sign or value or behavior of a function or derivative or integral. In finding an absolute extremum of a function on a closed interval, the student should show that all critical points as well as the endpoints of the interval were considered as candidates. See the scoring guidelines for 2010 AB6, 2010 BC3, 2009 AB1/BC1, 2009 Form B AB3/BC3, 2008 BC4, 2008 AB4/BC4, 2007 AB2/BC2, 2007 AB5/BC5 for examples.*

"Show the analysis (or work) that leads to your conclusion."

This reminder prompts the student to indicate the methods used.

Examples: *In finding an absolute extremum of a function on a closed interval, the student should show that all critical points as well as the endpoints of the interval were considered as candidates. In finding a slope value, the student should show how it was obtained. See the scoring guidelines for 2005 Form B AB2/BC2, 2004 Form B AB2, 2003 AB2, and 2003 Form B AB2 for examples.*

"Show the computations (or work) that leads to your answer."

This reminder emphasizes that it is particularly important for the student to show how a final numerical result was obtained.

Examples: *In finding an approximation to a value using a technique such as Euler's method, a Riemann sum or trapezoidal approximation, or a difference quotient, the student should show the basis of the computation, not just the final numerical result. See the scoring guidelines for 2010 AB2/BC2, 2010 BC5, 2010 Form B AB3/BC3, 2009 AB5, 2009 Form B AB6, 2009 Form B BC 4, 2008 AB2/BC2, 2008 Form B AB3/BC3 for examples.*

***FAQ:** Are there <u>any</u> computations that a student can perform with the calculator without need for showing intermediate computational steps?*

***Answer:** Yes. On Part A (the first two free-response questions), students are assumed to have a graphing calculator that can 1) graph a function, 2) numerically solve an equation, 3) numerically compute the value of a derivative at a point, and 4) numerically calculate the value of a definite integral. A student can freely use a calculator for any of these purposes without showing any intermediate work, as long as the student clearly indicates using mathematical language (not calculator syntax) how the calculator was used (referred to in the directions as the "setup"). With respect to the four capabilities just mentioned, this means: 1) labeling the function, the axes, and the scaling for a graph sketched from the calculator, 2) stating the equation that was solved, 3) stating the function and the point at which its numerical derivative was calculated, and 4) stating the definite integral that has been calculated. Note, however, that a graph reproduced from a calculator is not sufficient as the basis of a mathematical argument. (E.g., a graph obtained from a calculator is not sufficient justification for the existence of an extremum.) Although there are multiple calculator methods for solving an equation, it is recommended that students look at the graphs of functions to help orient themselves toward identifying the possible solutions or the number of solutions. This method provides students with a visualization of all the solutions, which makes finding the appropriate solution more efficient. On the graph screen, students should use "intersect" or "root/zero" commands. Students should avoid the use of tracing along a graph, which might not produce the required accuracy. Information from this graphical approach to solving an equation should be used before utilizing the calculator's numerical or symbolic "solver" commands.*

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\, dx$ may not be written as fnInt(X^2, X, 1, 5).

***Comment:** The terminology and notation of calculus provide the common language that students and teachers can use to communicate and share ideas. We might draw an analogy with standard language and regional slang. Calculator syntax (that varies with make and model) might make sense to those who use the same type of calculator, but could be foreign to someone using another type. However, it is expected that all students of calculus recognize and use the standard mathematical notation for objects studied in calculus.*

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

***Comment:** In general, answers that have been simplified are expected to have been simplified correctly. However, unless specifically instructed otherwise, it really is okay on the AP Exams to write* $2 + 3$ *instead of* 5, *or* $x + 5 + 2x - 3$ *instead of* $3x + 2$ *and other similar unsimplified expressions. The purpose of the exams is to assess a student's knowledge of calculus, not to score numerical or algebraic simplification skills. Nevertheless, if a computational result must be interpreted or used in a later part of the question, a student may find it useful to write that result in a simpler form. Thus, teachers may certainly demand more stringent simplification standards in their own AP classes.*

***FAQ:** Does a definite integral count as an unsimplified final numerical answer?*

***Answer:** No. Unless the question specified that the answer be reported as an integral expression without further evaluation, a student would be expected to compute the value of a definite integral.*

Comment: *With the use of calculators on the exam, it was necessary to adopt a rule to evaluate the accuracy of numerical answers reported in decimal form. The three decimal place standard has been used every year since 1995. Note that the standard can be overridden in a specific question. For example, in an application problem the student could be asked for an answer rounded to the nearest whole number. A recurring difficulty encountered in the scoring is the inappropriate application of this standard to all calculations. If a student rounds intermediate computations to only three places on the way to the final answer, this premature rounding may result in a failure to achieve the desired accuracy in that final answer. A useful calculator strategy for dealing with these intermediate results as accurately as possible is to store them. For example, the limits of integration on a definite integral might need to be computed using the calculator's graphical or numerical solver to find the intersection points of two graphs. Rather than rounding these values, a student should simply store the computed values as A and B (provided these particular letters are not used in some other way in the question). This would eliminate any need to retype several digits repeatedly while maintaining as much precision as possible. In writing the integral, the student should show these limits given to at least three decimal places, but in computing the integral, the student could use the stored values. Alternatively a student could write out the more precise numerical values of A and B, and then simply use the labels A and B as appropriate. See the scoring guidelines for 2009 BC3 and 2008 AB1/BC1 for examples.*

FAQ: *<u>MUST</u> answers reported in decimal form be rounded to only three decimal places?*

Answer: *Definitely not—reporting more accuracy is not penalized! The standard refers to the <u>minimal</u> accuracy expected in a final decimal answer. It should not be read as a requirement to round or truncate decimal answers, but rather to record decimal answers accurately to at least three places after the decimal point.*

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Comment: *This domain convention, common to most first-year courses in calculus, has been in effect for many years.*