Errata sheet for AP Calculus AB and AP Calculus BC

This document lists corrections and/or refinements made to the AP Calculus AB/BC Course and Exam Description since it was published in May of 2019.

Corrections to AP Calculus AB/BC as of September, 2019

The items listed below have been corrected in the online version of the CED. Teachers can print out the individual pages in order to update their printed CED binders.

- Instances of Mathematical Practice 2.B incorrectly included the word "symbolic." Note: this was not found in the main Mathematical Practices chart on p. 14. Pages where the skill incorrectly included the word "symbolic" were in Unit 1 on the Unit at a Glance (p. 32), Topic 1.1 (p. 35), Topic 1.2 (p. 36), Topic 1.3 (p. 37), Topic 1.4 (p. 38), on the Unit at a Glance for Unit 2 (p. 54), Topic 2.1 (p. 57), on the Unit at Glance for Unit 8 (p. 146), and Topic 8.6 (p. 154)
- In Topic 5.2 on p. 97, in Essential Knowledge FUN-1.C.1 when discussing the Extreme Value Theorem, there should be brackets around [a, b] and not parentheses.
- In Topic 5.2 on p. 97, there were two Essential Knowledge statements that were omitted from the CED. Specifically:
    - FUN-1.C.2: A point on a function where the first derivative equals zero or fails to exist is a critical point of the function.
    - FUN-1.C.3. All local (relative) extrema occur at critical points of a function, though not all critical points are local extrema.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~22–23 CLASS PERIODS (AB) ~13–14 CLASS PERIODS (BC)
CHA-1	**1.1 Introducing Calculus: Can Change Occur at an Instant?**	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.	
LIM-1	**1.2 Defining Limits and Using Limit Notation**	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.	
	1.3 Estimating Limit Values from Graphs	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.	
	1.4 Estimating Limit Values from Tables	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.	
	1.5 Determining Limits Using Algebraic Properties of Limits	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	1.6 Determining Limits Using Algebraic Manipulation	1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).	
	1.7 Selecting Procedures for Determining Limits	1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (e.g., Use the chain rule to find the derivative of a composite function).	
	1.8 Determining Limits Using the Squeeze Theorem	3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.	
	1.9 Connecting Multiple Representations of Limits	2.C Identify a re-expression of mathematical information presented in a given representation.	

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TOPIC 1.1

Introducing Calculus: Can Change Occur at an Instant?

Required Course Content

ENDURING UNDERSTANDING

CHA-1

Calculus allows us to generalize knowledge about motion to diverse problems involving change.

LEARNING OBJECTIVE

CHA-1.A

Interpret the rate of change at an instant in terms of average rates of change over intervals containing that instant.

ESSENTIAL KNOWLEDGE

CHA-1.A.1

Calculus uses limits to understand and model dynamic change.

CHA-1.A.2

Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero.

CHA-1.A.3

The limit concept allows us to define instantaneous rate of change in terms of average rates of change.

SUGGESTED SKILL

Connecting Representations

2.B

Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

SUGGESTED SKILL

Connecting Representations

2.B

Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

AVAILABLE RESOURCES

- Professional Development > **Definite Integrals: Interpreting Notational Expressions**
- AP Online Teacher Community Discussion > **How to "say" some of the notation**

TOPIC 1.2

Defining Limits and Using Limit Notation

Required Course Content

ENDURING UNDERSTANDING

LIM-1

Reasoning with definitions, theorems, and properties can be used to justify claims about limits.

LEARNING OBJECTIVE

LIM-1.A

Represent limits analytically using correct notation.

ESSENTIAL KNOWLEDGE

LIM-1.A.1

Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\lim_{x \to c} f(x) = R$.

EXCLUSION STATEMENT

The epsilon-delta definition of a limit is not assessed on the AP Calculus AB or BC Exam. However, teachers may include this topic in the course if time permits.

LEARNING OBJECTIVE

LIM-1.B

Interpret limits expressed in analytic notation.

ESSENTIAL KNOWLEDGE

LIM-1.B.1

A limit can be expressed in multiple ways, including graphically, numerically, and analytically.

TOPIC 1.3

Estimating Limit Values from Graphs

Required Course Content

ENDURING UNDERSTANDING

LIM-1

Reasoning with definitions, theorems, and properties can be used to justify claims about limits.

LEARNING OBJECTIVE

LIM-1.C

Estimate limits of functions.

ESSENTIAL KNOWLEDGE

LIM-1.C.1

The concept of a limit includes one sided limits.

LIM-1.C.2

Graphical information about a function can be used to estimate limits.

LIM-1.C.3

Because of issues of scale, graphical representations of functions may miss important function behavior.

LIM-1.C.4

A limit might not exist for some functions at particular values of x. Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from the left does not equal the limit from the right.

SUGGESTED SKILL

Connecting Representations

2.B

Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

ILLUSTRATIVE EXAMPLES

For LIM-1.C.4:

- $\lim_{x\to 0} \frac{1}{x^2} = \infty$
- $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.
- $\lim_{x\to 0} \sin\left(\frac{1}{x}\right)$ does not exist.
- $\lim_{x\to 0} \frac{1}{x}$ does not exist.

AVAILABLE RESOURCES

- AP Calculator Policy
- Classroom Resource > AP Calculus Use of Graphing Calculators
- Professional Development > Limits: Approximating Values and Functions
- Classroom Resource > Approximation

SUGGESTED SKILL

Connecting Representations

2.B

Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

AVAILABLE RESOURCES

- **AP Calculator Policy**
- Classroom Resource > **AP Calculus Use of Graphing Calculators**
- Professional Development > **Limits: Approximating Values and Functions**
- Classroom Resource > **Approximation**

TOPIC 1.4
Estimating Limit Values from Tables

Required Course Content

ENDURING UNDERSTANDING

LIM-1

Reasoning with definitions, theorems, and properties can be used to justify claims about limits.

LEARNING OBJECTIVE

LIM-1.C

Estimate limits of functions.

ESSENTIAL KNOWLEDGE

LIM-1.C.5

Numerical information can be used to estimate limits.

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~13–14 CLASS PERIODS (AB) ~9–10 CLASS PERIODS (BC)
CHA-2	**2.1 Defining Average and Instantaneous Rates of Change at a Point**	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.	
	2.2 Defining the Derivative of a Function and Using Derivative Notation	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems. 4.C Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).	
	2.3 Estimating Derivatives of a Function at a Point	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
FUN-2	**2.4 Connecting Differentiability and Continuity: Determining When Derivatives Do and Do Not Exist**	3.E Provide reasons or rationales for solutions and conclusions.	
FUN-3	**2.5 Applying the Power Rule**	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	2.6 Derivative Rules: Constant, Sum, Difference, and Constant Multiple	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
FUN-3 LIM-3	**2.7 Derivatives of $\cos x$, $\sin x$, e^x, and $\ln x$**	1.E Apply appropriate mathematical rules or procedures, with and without technology.	

TOPIC 2.1

Defining Average and Instantaneous Rates of Change at a Point

Required Course Content

ENDURING UNDERSTANDING

CHA-2

Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.

LEARNING OBJECTIVE

CHA-2.A

Determine average rates of change using difference quotients.

ESSENTIAL KNOWLEDGE

CHA-2.A.1

The difference quotients $\frac{f(a+h)-f(a)}{h}$ and $\frac{f(x)-f(a)}{x-a}$ express the average rate of change of a function over an interval.

CHA-2.B

Represent the derivative of a function as the limit of a difference quotient.

CHA-2.B.1

The instantaneous rate of change of a function at $x = a$ can be expressed by $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$, provided the limit exists. These are equivalent forms of the definition of the derivative and are denoted $f'(a)$.

SUGGESTED SKILL

Connecting Representations

2.B

Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

TOPIC 5.2

Extreme Value Theorem, Global Versus Local Extrema, and Critical Points

Required Course Content

ENDURING UNDERSTANDING

FUN-1

Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

LEARNING OBJECTIVE

FUN-1.C

Justify conclusions about functions by applying the Extreme Value Theorem.

ESSENTIAL KNOWLEDGE

FUN-1.C.1

If a function f is continuous over the interval $[a, b]$, then the Extreme Value Theorem guarantees that f has at least one minimum value and at least one maximum value on $[a, b]$.

FUN-1.C.2

A point on a function where the first derivative equals zero or fails to exist is a critical point of the function.

FUN-1.C.3

All local (relative) extrema occur at critical points of a function, though not all critical points are local extrema.

SUGGESTED SKILL

Justification

3.E

Provide reasons or rationales for solutions and conclusions.

AVAILABLE RESOURCES

- Classroom Resource > **Why We Use Theorem in Calculus**
- Professional Development > **Continuity and Differentiability: Establishing Conditions for Definitions and Theorems**
- Professional Development > **Justifying Properties and Behaviors of Functions**
- Classroom Resource > **Extrema**
- **On the Role of Sign Charts in AP Calculus Exams**

Applications of Integration

UNIT AT A GLANCE

Enduring Understanding	Topic	Suggested Skills	Class Periods ~19–20 CLASS PERIODS (AB) ~13–14 CLASS PERIODS (BC)
CHA-4	**8.1 Finding the Average Value of a Function on an Interval**	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	8.2 Connecting Position, Velocity, and Acceleration of Functions Using Integrals	1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems.	
	8.3 Using Accumulation Functions and Definite Integrals in Applied Contexts	3.D Apply an appropriate mathematical definition, theorem, or test.	
CHA-5	**8.4 Finding the Area Between Curves Expressed as Functions of x**	4.C Use appropriate mathematical symbols and notation (e.g., Represent a derivative using $f'(x)$, y', and $\frac{dy}{dx}$).	
	8.5 Finding the Area Between Curves Expressed as Functions of y	1.E Apply appropriate mathematical rules or procedures, with and without technology.	
	8.6 Finding the Area Between Curves That Intersect at More Than Two Points	2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.	
	8.7 Volumes with Cross Sections: Squares and Rectangles	3.D Apply an appropriate mathematical definition, theorem, or test.	
	8.8 Volumes with Cross Sections: Triangles and Semicircles	3.D Apply an appropriate mathematical definition, theorem, or test.	
	8.9 Volume with Disc Method: Revolving Around the x- or y-Axis	3.D Apply an appropriate mathematical definition, theorem, or test.	

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SUGGESTED SKILL

Connecting Representations

2.B

Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

TOPIC 8.6

Finding the Area Between Curves That Intersect at More Than Two Points

Required Course Content

ENDURING UNDERSTANDING

CHA-5

Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval.

LEARNING OBJECTIVE

CHA-5.A

Calculate areas in the plane using the definite integral.

ESSENTIAL KNOWLEDGE

CHA-5.A.3

Areas of certain regions in the plane may be calculated using a sum of two or more definite integrals or by evaluating a definite integral of the absolute value of the difference of two functions.